From Intensional Properties to Universal Support

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Abstract
A *factorial typology* is a set of grammars. We are not given the grammars directly, but must deduce them from the way that the posited constraints deal with the posited structures. How do we know that we have examined enough candidate sets to discriminate all the grammars that are allowed by our assumptions? This is the problem of finding a *universal support* for a typology. Without a universal support, we don't have the typology, and without the typology, many types of systematic claims about it must languish unjustified.

Here we show how the universal status of a proposed support may be established when we have exact descriptions of the types of optima allowed in the grammars. If a typology is factored into (intensional) ranking *properties* in the sense of Alber & Prince (in prep.), and if the property values are associated with (extensional) *characteristics* carried by optima, then a grammar as a combination of values is associated with a description of its optima as a conjunction of the characteristics associated with the values. If the descriptions thereby obtained uniquely denote single candidates, then the grammars cannot be further refined, and the support that produced the grammars must be universal.

This method of associating extensional characteristics with ranking patterns answers a much more general question: what do the languages of a typology look like? Since a typology is generated from a finite sample of candidate sets, we cannot in general be satisfied with remarking about the distribution of characteristics in the sample. We must use the grammars to project over the entire set of optima. The grammatical structure relevant to this enterprise is encoded in the ranking properties that combine to give the grammars.

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§0.0 Introduction

An OT system S is defined by specification of GEN_S and CON_S. The *typology* of S is the collection of all grammars admitted by that definition. A typology is always determined by a finite collection of candidate sets (csets), even when an infinite number of csets is admitted by GEN_S. An individual (*ranking*) *grammar* is the set of linear orders on CON_S that all produce the same collection of optima, the same extensional *language*. Every ranking grammar is characterizable as an ERC set; the rankings that belong to it are the ***l***inear ***e***xtensions of the ***g***rammar's ERC set, termed 'legs'. A *support* for a single grammar is a collection of csets that suffices to delimit all the legs of the grammar when the appropriate candidates are selected as optima. A *universal support* for a typology is a collection of csets that suffices to deliver all of its grammars when every possible combination of optima is considered. A *minimal* universal support is a universal support from which no cset can be removed without destroying its universality. These concepts and terminology, which develop and further specify those of Prince & Smolensky 1993/2004 (P&S), are from Prince 2014, 2014-15, 2015, Merchant & Prince (in prep.), and Alber & Prince (in prep).

The logic of OT analysis requires not only that the system S under scrutiny be defined by spelling out $\langle GEN_S, CON_S\rangle$, but that the typology claimed for S be derived from a valid universal support. As long as each cset contains all of its possible optima, the source of failure will be omission of at least one necessary cset. The penalty for failing to employ a valid universal support in this case is that the claimed typology is *coarser* than the real one. This means that while the distinctions between grammars that are established by the insufficient support are veridical, there are yet further distinctions that must be drawn to arrive at the real typology, which is *finer* than the generated one. Some of the grammars of the coarser typology turn out to be amalgams of two or more grammars from the actual typology of the system S as defined. In particular, some ranking relations that are left open in grammars of the coarser typology must be specified, splitting the grammars, in order to arrive at the correct typology.

An instructive example of a too-coarse typology constructed from an insufficient support is provided by the Basic Syllable Theory (BST) as presented in chapter 6 of P&S. These authors define $\langle GEN_{BST}, CON_{BST}\rangle$ and go on to analyze 9 grammars as the BST Typology. As Riggle (2004) was the first to announce, the actual number of grammars is 12. The shortfall is due to the omission of an input that contains a consonant that cannot be faithfully syllabified under the limitations of GEN_{BST}. P&S consider two csets: one from /V/, the other from /CVC/. The first gives rise to the triple fates of vowels that cannot be faithfully syllabified into syllables shaped $[_\sigma CV]$: faithful reproduction, epenthesis of C into onset, deletion of the vowel. The second gives rise to the triple fate of consonants that cannot be faithfully syllabified into $[_\sigma CV]$ syllables, but *which can be faithfully syllabified* into syllables $[_\sigma CVC]$: faithful reproduction, epenthesis of V, deletion of the postvocalic C. These interact freely, and $3\times3 = 9$.

Missing is an input like /C/, for which GEN_{BST} provides no faithful parse. GEN_{BST} requires that all syllables contain a V, disallowing the structure $[_\sigma C]$. The two admitted optimal outputs from /C/

involve either epenthesis of a vowel to support the input C in the syllable [$_\sigma$CV] or deletion of the C to arrive at the empty string, which lacks syllables altogether and therefore slips past all of their defining requirements. Both candidates fully satisfy all markedness constraints, so that the choice between them is made purely on grounds of faithfulness: f.max$\gg$f.depV yields the epenthetic optimum, and f.depV$\gg$f.max yields the deletional, empty output. It follows that *every* grammar in the typology crucially ranks these two constraints with respect to each other, because an OT grammar finds an optimum in every cset.

The effect is therefore felt on those three grammars of the coarser P&S typology in which no ranking holds between f.depV and f.max. These are the 3 grammars in which /CVC/ is faithfully reproduced as [$_\sigma$CVC]. P&S note that these coda-allowing grammars meet the condition that *both* f.max and f.depV dominate m.NoCoda. (There are three of them, because they freely combine with the rankings derived by the 3 possible optima from /V/). The input /C/ — or /CCVVC/ from Riggle (2004:109) — will further split each of these coda-allowing grammars, based on their treatment of underlying C that cannot be faithfully syllabified in any grammar, which always forces a choice between epenthesis and deletion.

The three P&S coda-allowing grammars each contain legs λ_1 and λ_2 which have the following form, where the material in the sequentially corresponding sequences "…" is assumed identical across the legs.

λ_1 = …f.max…$\gg$ …f.dep… $\gg$ …m.NoCoda…

λ_2 = …f.dep… $\gg$ …f.max…$\gg$ …m.NoCoda…

In each of the P&S candidate sets, these legs produce the same optimum. In particular, CVC→[$_\sigma$CVC] is optimal under both. When /C/ is brought into the picture, however, they produce different outputs, with λ_1 sponsoring C→[$_\sigma$CV] and λ_2 sponsoring C→∅. Because these candidates are admitted as competitors by GEN_{BST} and because they do not have identical violation profiles, it can only be that the P&S typology is too coarse for the BST as defined, and that it does not rest on a valid universal support. The P&S typology accurately characterizes a somewhat different OT system that we might call CST, where GEN_{CST} restricts inputs to allow CC only between vowels. CST provides a good approximation to BST, and in many languages the difference will not be noticed. But as we've seen, the coda-allowing grammars in CST contain legs that do not agree on the optimum of certain csets defined in BST. From the point of view of the BST, these CST grammars are not grammars but mere collections of rankings. They lack the key grammar-defining property that we can call *uniform selection*, whereby all linear orders belonging to the grammar select the same extensional language.

Uniform selection gives us a way to establish the universality of a support for the typology of a system S. If we can show that each grammar derived from a given support selects a single violation profile as optimal in *every* candidate set admitted by GEN_S, then we can be sure that no grammar submits to further refinement: the support must be universal. On the face of it, this seems a burdensome requirement, inviting search of all grammars and all candidate sets, but it takes on a different character in the context of the Classification Program of Alber & Prince.

Alber & Prince (in prep.) analyze the ranking structure of the grammars of a typology into a set of (intensional) *properties*. Each property takes on a set of mutually exclusive *values*, where each value is a ranking condition. The grammars of the typology, understood as ERC sets that delimit sets of rankings as their legs, are generated in their entirety by selecting values from the properties, with the possibility of scope restrictions that limit the freedom of combination. In the stress system nGX, examined in detail below, the property "FTyp.ia/tr" requires a choice between Iamb$\gg$Trochee and Trochee$\gg$Iamb. Every grammar must choose one or the other of these. Similarly, the property "Pos.L/R" requires a choice between AFL$\gg$AFR and AFR$\gg$AFL, where "AFL" abbreviate the Generalized Alignment constraint commonly called "All Feet Left" and similarly for AFR, with exchange of chirality. Alber & Prince show that choosing values from these plus two other additional properties generates the entire set of ERC grammars obtained from their support for the nGX typology.

These *properties* are intensional in that they determine grammars, but they have extensional consequences, determining the structural *characteristics* that appear in optima. For example, as we will show below, in an optimal output of grammars that satisfy the nGX property value FType.ia, *every* binary foot in a word is iambic; in those that satisfy FType.tr *every* binary foot is trochaic. This is the extensional force of the intensional property.

It is worthwhile pausing to ponder the conceptual chasm between the intensional property and its extensional effects. A property like FType.ia/tr or Pos.L/R is about the ranking structure of grammars and enforces a relation between certain constraints. These constraints are not defined to dictate the shape of entire output forms: they merely accumulate penalties for certain structural configurations, often quite local, that occur within outputs. They do not say things like 'every binary foot is iambic in optima', or 'the unary and binary feet in optima are disposed in such-and-such a way'. On the extensional side, the various structural patterns appearing in optima show a distribution that may be quite distantly related to the types of items and relations monitored by the constraints. For this reason, we enforce a terminological distinction between intensional *properties* and extensional *characteristics*. The nature of these characteristics and their distribution emerges from the functioning of the grammar. For any OT system S, this involves the interplay of the constraints of CON_S, the candidate sets admitted by GEN_S, and the definition of optimality. Analysis rather than onomasty is required to establish the intensional-extensional connection.

When we understand the extensional characteristics associated with the intensional properties that generate a typology, we have a full description of the linguistic structure imposed by the grammars of the typology. This is essential to understanding what the typology says about linguistic form. A valid universal support delivers the entire set of grammars in a typology, but it is never more than a finite sample of what is typically an infinite set. The distribution of extensional characteristics in a finite support may be suggestive of the broader pattern, but the mere finitude of the sample typically disallows secure generalization. The grammars must be examined, because they define the behavior of *every* cset. And the properties define the grammars.

From a property set for a typology, a *classification* in the terminology of Alber & Prince, we can derive a complete description of the typology's extensional characteristics if we have a full analysis of the extensional consequences of the properties. A grammar is specified as a set of property values, and these induce a set of characteristics which completely delimits the extensional structure of the optima for that grammar. Just as the grammar emerges from conjoining the property values, so does the description of its optimal forms emerge from conjoining the descriptions of characteristics associated with those values.

Aside from whatever desirability inheres in relating the grammars of a typology to their predictions about linguistic form, there is a further use for the full extensional interpretation of the intensional properties: it can verify that the typology was generated from a valid universal support. From a proposed support, we can mechanically calculate a set of grammars, which may be classified into a set of properties and their values. The validity and completeness of a proposed classification may easily be checked by simply running through the sets of value choices and verifying with the entailment algorithm of Prince 2002a:31ff that the ERC sets so derived are in a mutually entailing relation with the grammars produced by calculation; or the check may call on FRed (Brasoveanu & Prince 2011) to show that the grammars produced by the property set are identical in MIB or SKB form with those produced by the support.

Verifying that the support is universal requires further analysis. A claimed support is authentically universal if the grammars it induces cannot be further refined. Refinement requires splitting a putative grammar by determining that it contains distinct legs that produce distinct optima from a single input.

The close association between properties and extensional characteristics allows us to pursue the universality question in the extensional domain. When a typology is analyzed as a set of properties, each of its grammars is equivalent to a conjunction of (intensional) property values. These values entail extensional characteristics. If every conjunction of property-value-derived *characteristics* yields the description of a *single* optimal candidate for each input admitted by GEN_S, then we can be sure that the grammars associated with the property values cannot be refined. (In the fully general case, which we will not encounter here, the extensional description can admit multiple candidate instantiations when they all have the same violation profile.) From a grasp of the intensional and extensional structure of the typology, we can prove — or in the case of non-uniqueness, disprove — the universality of a proposed support. In short, knowing how every grammar behaves extensionally tells us whether we know of every grammar.

We carry out this program for the foot-based prosodic system nGX as analyzed by Alber & Prince. The bulk of our argument develops a detailed analysis of the extensional characteristics associated with property values. We then show that each choice of values yields a description that delimits a single candidate for each distinct input. We conclude by using this value-based description to characterize the possible universal supports of nGX.

§0.1 The system nGX and its properties

From Alber & Prince (in prep.), we have the following specification of the system nGX, which recognizes various aspects of stress patterning that do not depend on the presence of a main stress and which does not impose any left-right asymmetries in metrical patterning.

(1) **$\mathbf{GEN_{nGX}}$**

a. Inputs are strings of atomic units, representing syllables.
b. An input is associated with outputs of exactly the same length in syllables.
c. An output consists of a single Prosodic Word.
d. A Prosodic Word consists of feet and syllables.
e. A Foot consists of one or two syllables.
f. A syllable may belong to at most one foot.
g. A Foot has a unique head.
h. A Prosodic word has at least one foot.
i. The output set from an input contains every parse admitted by these requirements.

Certain characteristics of output forms of nGX are fixed by GEN_{nGX} and others are left open to vary between candidates. The units of nGX structure are easily and briefly enumerated: syllable, Foot, Prosodic Word. A syllable is parsed into a foot or not, in which case it is termed an *unparsed* syllable. A foot, monosyllabic or bisyllabic, contains a distinguished syllable, the head. The Prosodic Word is free to contain feet in any non-overlapping, non-recursive disposition, so long as at least one is present. To uniquely identify a specific output form, we need only indicate its length in syllables, where in the syllabic string each of its feet begins and ends, and which syllables have the status of head-of-foot. This description is unambiguous because there is literally nothing else to specify. This observation makes up in usefulness what it lacks in profundity, and is therefore worthy of recognition.

(2) **Remark**. **Unambiguous Description of nGX forms**. An output form of nGX is uniquely identified by its length, the location of its feet with respect to the syllable string, and the location of the heads of the feet.

To spell out CON_{nGX}, we introduce some notation. OTWorkplace (Prince, Tesar, and Merchant, 2007-2015) provides a convenient string-based spelling system for the parses of nGX and similar prosodic systems:

o	syllable not parsed into a foot (unparsed syllable)
u	nonhead of a foot
X	head of foot
-	edge of foot or unparsed syllable

This is easily seen to accord with the requirements of Remark (2), and therefore can be used to refer without fear of ambiguity to the hierarchical structures sanctioned by GEN_{nGX}. In addition, we will write 'f' for 'foot of any kind,' 'F' for 'binary foot,' 'U' for 'unary foot,' and 'σ' for 'syllable of any kind'. In constraint definitions, we use $\lessdot$ to mean 'precedes' and '$\gtrdot$' to mean ‘follows’. With these notations, the constraints of nGX may be defined as follows, where * is the OT star operator returning the number of matches in a candidate to the pattern specified after it.

(3) **$\mathrm{CON}_{\mathrm{nGX}}$**

- Parse-σ (Ps) *o
- Iamb *-X
- Troch *X-
- AFL *{σ,f}: σ ⋖ f
- AFR *{σ,f}: σ ⋗ f

The definitions of AFL and AFR are derived from those of Hyde (2012:803, 2015:22). We elide reference to the Prosodic Word as the domain of alignment, since it is the only domain admitted by $\mathrm{GEN}_{\mathrm{nGX}}$ that contains σ and f. On this approach, AFL accumulates a penalty for each pair {σ,f} in an output where σ precedes f. This provides an overall score for a form that effectively sums the distance in syllables of each foot f from the left edge of the Prosodic Word, which is exactly the intention of McCarthy & Prince (1993) for evaluation by this constraint, as first suggested by Robert Kirchner (p.c.). AFR does the same, in mirror image.

The name of the system acronymically encodes its key properties. The definitions of Iamb and Troch are ***n***ew, in that they penalize feet by placement of the head: Iamb penalizes head-initial feet, Troch head-final feet. The effect is that both penalize unary feet. In some earlier conceptions, the constraints penalized only -uX- as non-trochaic and -Xu- as non-iambic, treating unary feet as being *both* iambic and trochaic. Positioning of feet is done by ***G***eneralized Alignment (McCarthy & Prince 1993). All outputs consist of Prosodic Words with at least one foot: hence the suffixal ***X***.

With $\langle \mathrm{GEN}_{\mathrm{nGX}}, \mathrm{CON}_{\mathrm{nGX}} \rangle$ defined, a set of 12 grammars, listed in Appendix 1, may be generated from a support that consists of two complete csets with inputs of length 3σ and 4σ. The same 12 may be generated from an input of length 5σ. This replication hints that 12 might be the actual number of grammars in the typology, and that they might be exactly the ones we have generated. Our goal is to show conclusively that this is true, and that both supports are universal.

Observe that we are not trying to show that the extensional languages of nGX are those of the world around us. We execute the prior task of determining what the grammars of nGX actually are, free of all heuristics, speculation, and unjustified belief. Our method securely connects the grammars of nGX with the unbounded extensional languages that they predict, providing the basis for further empirical and theoretical development.

Alber & Prince analyze the set of 12 grammars into 4 properties. (We resist calling it the ‘typology of nGX’ until we have shown that it was generated from a universal support.) The terminology and notation is described immediately below the table of properties given here. The content of the properties is the subject of sections §1-4. A property value holds of every leg in a grammar that ‘satisfies’, ‘meets’, or ‘falls under’ that value.

(4) **Properties of nGX**

Name	Values	Ercish Form: Ps . Ia Tr . AFL AFR
FTyp.ia/tr	Iamb <> Troch	e.WL.ee / e.LW.ee
Pos.L/R	AFL <> AFR	e.ee.WL / e.ee.LW
Mult.D/sp	Ps <> ⟨Fsub, Adom⟩	W.(eL/Le).LL / L.(eW/We).WW
Un.X/o	Ps <> ⟨Fdom, Adom⟩	W.LL.LL / L.WW.WW

Terminology and notation. A property has a name N followed by the names of its values a and b, thus N.a/b. The names used here include the following mnemonically-chosen abbreviations.

(5) **Properties and Values**

Prop	Value	Abbreviates	
Ftyp	ia	foot type	iambic
	tr		trochaic
Pos	L	position (of feet)	left
	R		right
Mult	D	multiplicity (of feet)	dense
	sp		sparse
Un	o	unary constituents	unparsed syllable
	X		unary foot

The notation ⟨A,B⟩ <> ⟨C,D⟩ abbreviates a property with the two (contradictory) values given by reversal of the domination relation, accompanied by the appropriate interpretation of the mentioned sets of constraints. These interpretations are:

1. A|B ≫ C&D: ‘either A or B dominates both C and D,’ and
2. C|D ≫ A&B: ‘either C or D dominates both A and B.’

The relation between the values follows the ERCish treatment of aggregates of constraints: disjunction of the dominators, conjunction of the subordinates. Recall that the Elementary Ranking Condition associated with a vector on {W,L,e} asserts that *every constraint assessing L is dominated by some constraint assessing W* (Prince 2002a,b). The example may be represented as an ERC (fragment) WWLL and its opposite LLWW, where the constraints are listed in the arbitrary order A,B,C,D. The values of the second ERC are obtained by applying the rules of negation in ERC logic to the first. These are: ¬W=L, ¬L=W, ¬e=e. See Prince 2002a for detailed discussion and analysis.

Each property stated above bifurcates the typology of nGX into those grammars that satisfy one value and those that satisfy the other. Thus, the ‘scope’ of each property — the set of grammars to which it is relevant —is the entire typology. In the case of Un.o/X, there is a natural narrowing of scope which simplifies the property. When limited to Mult.D grammars, it becomes Ps<>Fdom, which sponsors opposing ERCs W.LL.ee / L.WW.ee. Alber & Prince refer to the phenomenon of limited scope as 'mootness', since the distinction made by the property is moot outside its scope. We show how this works at the end of §4.

Property values classify grammars by shared ranking requirements. Properties implicitly define classes of constraints as well: for example, FTyp recognizes {Iamb, Trochee} as a significant class. The notation Fdom, Fsub allows us to generalize over the members of the class F = {Iamb, Trochee}, referring to whichever of these is dominant (Fdom) or subordinate (Fsub) in the legs of a grammar. Similarly, A = {AFL, AFR} is identified as class of constraints by its participation in the property Pos.L/R.

For convenience of reference, we assume a fixed but arbitrary sequencing of the CON_{nGX} constraints in ERCS: Ps, Ia, Tr, AFL, AFR. We use dots to separate the constraints into the classes {Ps}, F = {Iamb, Troch}, A = {AFL, AFR}. This allows us to abbreviate ERCs as e.g. W.eL.Le, which represents Ps ≫ Tr & AFL, "Ps dominates both Tr and AFL."

§0.2 Mode of Argument

A grammar is specified by choice of property values. With properties from the inventory (5), we have grammars such as sp.ia.R.o, D.ia.L.o, D.tr.R.X, where each dotted slot in the sequence cites the name of a property value. Each value provides an ERC that holds of the grammar so defined. For example, Pos.L contributes the ERC e.ee.WL ("AFL≫ AFR"), FTyp.ia contributes e.WL.ee ("Iamb≫Troch"), and in accord with those choices, Mult.D contributes W.eL.Le ("Ps ≫ Fsub & Adom" ⇒ "Ps≫ Troch & AFL").

We investigate the extensional content of the properties by examining grammars that satisfy a certain property value or values. The same mode of argument is employed repeatedly. We analyze a competition between two candidates, call them **z** and **φ**, which are constructed so as to be in an informative relation. Competitors **z** and **φ** will typically differ in only one respect, thereby isolating the violation penalties contributed by the distinction between them. We determine the ERC **z**~**φ** by inspecting their relative performance on the constraints. Then we show that this ERC is entailed by the property or properties under consideration, employing the familiar inferential system of ERC logic (Prince 2002a, b). This establishes that **z** is *better than* **φ** in the grammar, which we write as '**z**≻**φ**'. If **z**≻**φ** in a given grammar, then **φ** cannot be optimal under the conditions that the grammar meets, because there is always a better candidate, namely **z**. Recall that **z**≻**φ** is a strict order relation defined with respect to a specific grammar, and means that, in every leg λ of that grammar, λ selects **z** from the set {**z**,**φ**}. If a grammar satisfies the ERC **z**~**φ**, where **z** and **φ** differ in violation profile, we are guaranteed that **z**≻**φ**.

The force of the argument is that *no* candidates of the form of **φ** can be optimal, and by choosing our **φ**'s and **z**'s properly, we can rule out all structural types except the one that mirrors the extensional content of the property we are examining. This is the one hammer we need to smite all the suboptima.

This is a bounding argument, of a type generally familiar from the OT literature. In the ordering of candidates imposed by the property value(s) under discussion, we show that **z** *bounds* **φ**, eliminating **φ**'s chances for optimality. This form of argument is commonly used to establish *harmonic bounding*, which holds in every ranking. (P&S 1993/2004:116-8, 209-212, Samek-Lodovici 1992). We make use throughout of a generalized form, focusing on the subset of rankings admitted by a property value or values, taking advantage of the resources of ERC logic. P&S (p. 262-4) prove aspects of their Onset Theorem and Coda Theorem (stated p.113-4) with similar arguments. Prince 2006 makes extensive use of ERC-logic entailment relations to establish properties of OT systems.

Certain of our conclusions require only harmonic bounding arguments, independent of the properties. These involve the distribution of unary feet in optima. From harmonic bounding alone we can show that an optimum must have at least one binary foot if it has enough syllabic material to support one: put another way, the feet of an optimal output of 2 syllables or greater in length cannot all be unary. To illustrate the character of a bounding argument, and to get a start on the characterization of extensional effects of the constraint system, we develop the argument here. We build up to the desired result through a sequence of two more specific lemmas, each established by a harmonic bounding argument. We use the U, F, and σ notation from above; foot edges, when invoked, are indicated by parentheses. We write |**α**| for the length in syllables of the form **α**. For any constraint C, we write C(**q**) for the numerical penalty assigned by C to candidate **q**, and thus an expression like C(**q**) $<$ C(**z**) declares that one number is less than another.

(6) **Lemma**. No optimal output contains a sequence UU.

Proof. Consider any form **φ** = […UU…] and a competitor **z** = […F...] which is exactly like **φ** in every respect except that the syllables parsed UU in **φ** are parsed as F in **z**. To construct the ERC **z**~**φ**, we compare C(**z**) and C(**φ**) numerically over every constraint $C \in CON_{nGX}$.

z = […F…], **φ** = […UU…]		
z~φ	Comparison of violation values	Rationale
e	Ps(**z**) = Ps(**φ**)	The status of syllables as footed or unfooted is the same in both.
W	Iamb(**z**) < Iamb(**φ**)	UU contributes 2 violations to Iamb(**φ**). If F is iambic, it contributes no violations to Iamb(**z**); if F is trochaic, it contributes 1 violation to Iamb(**z**). All other feet are the same in both competitors.
W	Troch(**z**) < Troch(**φ**)	The same reasoning applies mutatis mutandis.
W	AFL(**z**) < AFL(**φ**)	UU = (σ)(σ) and F=(σ σ). AFL sees UU as (σ (σ and F as (σ σ, where the left parentheses denote edges which, when non-initial, contribute violations to the AFL score. The UU structure in **φ** contains one more AFL-relevant foot edge than the F structure in **z** and that extra edge in **φ** is non-initial, guaranteeing at least one additional violation. All other feet are in the same positions in both.
W	AFR(**z**) < AFR(**φ**)	The same reasoning applies mutatis mutandis to UU = σ) σ) and F = σσ).

Given these numerical relations, the resulting ERC **z**~**φ** is e.WW.WW, indicating that **z** harmonically bounds **φ**. No form containing a sequence UU can be optimal. ☐

(7) **Lemma**. No optimal output contains a sequence Uo or oU.

Case 1**.** Let **φ** = […Uo…]. Consider a competitor **z** = […F...] which is exactly like **φ** in every respect except that the material parsed Uo in **φ** is parsed as F in **z**. To construct the ERC **z**~**φ**, we compare **z** and **φ** over every constraint.

$\mathbf{z}$ = […F…], $\boldsymbol{\varphi}$ = […Uo…]		
z~φ	Comparison of violation values	Rationale
W	$Ps(\mathbf{z}) < Ps(\boldsymbol{\varphi})$	The form **z** has one fewer unparsed syllable than **φ**.
e/W	$Iamb(\mathbf{z}) \leq Iamb(\boldsymbol{\varphi})$	If F is trochaic, we trade one non-iambic foot for another, and the scores are equal. If F is iambic, **z** has one less non-iambic foot than **φ**.
W/e	$Troch(\mathbf{z}) \leq Troch(\boldsymbol{\varphi})$	The same reasoning applies mutatis mutandis.
e	$AFL(\mathbf{z}) = AFL(\boldsymbol{\varphi})$	From the point of view of AFL, Uo = (σσ and F = (σσ, contributing equal numbers of violations to the AFL score.
W	$AFR(\mathbf{z}) < AFR(\boldsymbol{\varphi})$	From the point of view of AFR, Uo = σ)σ and F = σσ). Since the cited foot edge in Uo in **φ** is one syllable farther from the end of the form than the cited edge in F, it contributes one more violation to the AFR score than F in **z**.

The resulting ERC **z**~**φ** takes the form W.We.eW or W.eW.eW, depending on whether F is iambic or trochaic. Thus **z** harmonically bounds **φ**. No form containing Uo can be optimal.

Case 2. Let **φ** = […oU…] and **z** = […F…]. The reasoning here is the same as in Case 1, except that the roles of AFL and AFR are interchanged. The resulting ERCs take the form W.We.We or W.eW.We, depending on whether F is iambic or trochaic. Once again, **z** harmonically bounds **φ**. No form containing a sequence oU can be optimal. □

(8) **Proposition**. **Binarity**. No optimal output longer than 1σ lacks binary feet.

Proof. Let $\boldsymbol{\varphi} \in \{U,o\}^+$ where $|\boldsymbol{\varphi}|$ is 2 syllables or greater. From GEN_{nGX}, we have it that **φ** contains at least one foot. Consider some such form **φ** = […U…], where at least one of the stretches "…" is nonempty. U must be immediately followed or immediately preceded by a syllable to its left or by a syllable to its right. That neighboring syllable may be parsed as a unary foot, so the cited U is in a configuration UU. Or the neighboring syllable may be unparsed, so that cited U is in a configuration Uo or oU. There are no other cases. From Lemma (6) and Lemma (7), we know that no optima contain any of these configurations. Since GEN_{nGX} requires the presence of a foot in outputs, and since no $\boldsymbol{\varphi} \in \{U,o\}^+$ with $|\boldsymbol{\varphi}| \geq 2$ is optimal, the only optimal forms of length 2 syllables or longer have at least one binary foot. □

We conclude by noting a feature of ERC logic that we will take advantage of when convenient. In Boolean reasoning, if the antecedent is false, there is no need to worry about the truth value of the consequent: the same holds coordinatewise in ERC logic when an antecedent coordinate is L. Recall that entailment from one ERC to another depends on two rules of inference, L-retraction and W-extension (Prince 2002:5-7). To obtain $\alpha \vDash \beta$, where α, β are logically nontrivial in that they each contain both W and L, it is necessary and sufficient that in each coordinate *k* we have $\alpha[k] \leq \beta[k]$, where the comparative values are arrayed along an abstract scale $L<e<W$. The rule of W-extension says that if the consequent coordinate value $\beta[k]$ = W, then $\alpha[k]$ may be anything.

The rule of L-retraction says that if antecedent α[*k*] = L, then β[*k*] may be anything. The upshot is that in arguing α⊨β coordinatewise, we can ignore those coordinates *k* where α[*k*] has L, because nothing in β[*k*] can obstruct the entailment relation, by L-retraction.

(9) **Remark**. *Ex Falso Quodlibet*. For ERCs α and β, if α[*k*] = L, then in arguing α⊨β by checking relations between values in the coordinates, we need not check the value of β[*k*].

We will employ this handy shortcut on a couple of occasions below, to simplify the calculations when they threaten to distract.

§0.3 Conspectus of the Argument

Unaries (§0.2)

● Lemma (6). No optimal output contains a sequence UU.
● Lemma (7). No optimal output contains a sequence Uo or oU.
■ Proposition (8). **Binarity**. No optimal output longer than 1σ lacks binary feet.

FTyp.ia/tr (§1)

■ Proposition (10). **Uniformity of Foot Type**. All the binary feet of an optimal output have the same type, iambic or trochaic.

Pos.L/R (§2)

● Lemma (11). Under Pos.L, no optimal output contains FU. Under Pos.R, no optimal output contains UF.
● Lemma (12). No optimal output contains a unary foot displaced from the dominant edge.
► Corollary (13). One U. No optimal output contains more than one U.
● Lemma (14). No optimal output under Pos.L contains the sequence oF. No optimal output under Pos.R contains the sequence Fo.
■ Proposition (15). **Positioning of Feet in Optima**. In a grammar satisfying Pos.L, any optimal output of length 2 or greater must be of the form [(U)$F^n o^k$], $n \geq 1$, $k \geq 0$. In any grammar satisfying Pos.R, any optimal output of length 2 or greater must be of the form [$o^k F^n$(U)].

MULT.D/sp (§3)

● Remark (18). Shrift for the Short. Optimal outputs of length 1 are of the form [U]. Optimal outputs of the length 2 are of the form [F].

Mult.sp

● Lemma (19). Under Mult.sp = ⟨Adom, Fdom⟩≫Ps, no optimal output contains FF.
● Lemma (20). Under Mult.sp, no optimal output contains UF or FU.
■ Proposition (21). **The shape of the sparse**. In grammars satisfying Mult.sp = ⟨Adom, Fsub⟩≫ Ps, all optima of length 2 or greater take the form [Fo^k] under Pos.L and [o^kF] under Pos.R, $k \geq 0$.

Mult.D

● Lemma (22).Under Mult.D, no optimal output contains oo.

● Lemma (23). Under Mult.D, no optimal output contains both U and o.

■ Proposition (24). **The shape of the dense**. In grammars satisfying Mult.D = Ps≫ ⟨Adom,Fsub⟩, all optimal outputs of length 2 or greater take the form [F^n(o)] or [(U)F^n] under Pos.L and [(o)F^n] or [F^n(U)] under Pos.R, $n \geq 1$.

Un.o/X (§4)

Un.X

● Lemma (26). Un.o/X and Sparseness. Sparse grammars cannot assume the value Un.X. Sparse grammars must assume the value Un.o.

● Lemma (27). Under Un.X, no optimal output contains o.

■ Proposition (28). **The shape of Un.X.** Under Un.X = Ps ≫ ⟨Fdom, Adom⟩, all optimal outputs of length 2 or greater take the form [(U)F^n] under Pos.L and [F^n(U)] under Pos.R, $n \geq 1$.

Un.o

● Lemma (29). Under Un.o, no optimal output of length 2 or greater contains U.

■ Proposition (30). **The shape of Un.o**. In grammars satisfying Un.o, optima are of the form [$F^n o^k$] under Pos.L and [$o^k F^n$] under Pos.R, $n \geq 1$, $k \geq 0$.

Extensional Characteristics of the Optima of nGX (§5)

Theorem (32). **Optimal Outputs of nGX**. The optimal outputs of nGX of length greater than or equal to 2 syllables are drawn from the following patterns. Within each schema, F is uniformly iambic or uniformly trochaic, and $n \geq 1$, $k \geq 0$.

Mult/Un values	Pos.L	Pos.R
Mult.sp, (Un.o)	Fo^k	$o^k F$
Mult.D, Un.o	F^n(o)	(o)F^n
Mult.D, Un.X	(U)F^n	F^nU

Universal Supports for nGX (§6)

Theorem (33). **Universal Supports for nGX**. Any collection of candidate sets that delivers the 12 grammars obtained from the properties (4) is a universal support for nGX.

● Lemma (34). The Long Supports. Any odd-length input of 5 or more syllables provides a universal support for nGX.

● Lemma (35). Failure of the Even. No even length provides a universal support.

● Lemma (36). The support with two inputs, one of length 3, the other of length $2m$, $m \geq 2$, is universal and minimal**.**

Theorem (37). **Minimal Universal Supports for nGX**. The minimal universal supports for nGX are (1) the csets from an input of length 3 and an input of length $2m$, $m \geq 2$**,** and (2) any single cset from an input of odd length 5 syllables or greater. There are no others.

§1 FTyp.ia/tr: Iamb <> Troch

We begin by establishing the extensional effects of the FTyp.ia/tr property.

(10) **Proposition**. **Uniformity of Foot Type**. All the binary feet of an optimal output have the same type, iambic or trochaic. The binary Foot type of the entire output form is determined by FTyp (Fdom ≫ Fsub).

Proof. Assume FTyp.ia = e.WL.ee, so that Fdom = Iamb. Consider any output **φ** that contains a binary foot F̄ of the subordinate trochaic type, so that **φ** = [… F̄ …]. Now consider an output **z** = […F…] which is exactly the same as **φ** except that the cited trochaic foot F̄ is replaced by iambic F. Claim: **z**≻**φ** in any grammar meeting FTyp.ia. To show this, we construct the ERC **z**~**φ**. Observe that both **z** and **φ** fare exactly the same on the constraints Ps, AFL, AFR. This leaves only Iamb and Troch to distinguish foot type, as claimed.

The form **φ** has one more non-iambic foot than **z**, so that Iamb(**z**) < Iamb(**φ**). On Troch, we have Troch(**z**) > Troch(**φ**), because **z** has one more non-trochaic foot than **φ**. We therefore have the following ERC:

	Ps	Iamb	Trochee	AFL	AFR
z~**φ**	*e*	W	L	*e*	*e*

We abbreviate this as **z**~**φ** = e.WL.ee.

Any grammar satisfying FTyp.ia = e.WL.ee meets this condition. The ERC **z**~**φ** is trivially entailed by FType.ia, and we have **z**≻**φ**. This means that **φ** cannot be optimal in any grammar satisfying FType.ia, because there is always a competitor better than **φ**. Any form like **φ** that contains even a single trochaic foot F̄ has a competitor that betters it by virtue of lacking that foot. Whether the competitor is itself bettered by something else is immaterial to deducing the fate of **φ**. Therefore, in grammars under FType.ia, no trochaic binary foot appears in an optimum.

The argument holds, mutatis mutandis, for FTyp.tr when we simply interchange the roles of Iamb and Troch.

Thus no form with any subordinate-type binary feet is optimal, and each optimum must be chosen from among those with only dominant-type binary feet, as claimed. □

§2 Pos.L/R: AFL <> AFR

We now establish a number of facts about the positioning of feet in nGX.

(11) **Lemma**. Under Pos.L, no optimal output contains FU. Under Pos.R, no optimal output contains UF.

Proof. Assume that the grammar under consideration satisfies Pos.L = AFL≫AFR. Let **φ** = […FU…] and consider a competitor **z** = […UF…], identical to **φ** in every respect except for the parsing of the three syllables dominated by the cited F and U nodes. We calculate the ERC **z**~**φ**. Observe first that **φ** and **z** fare identically on the constraints Ps, Iamb, and Trochee. This gives us **z**~**φ** = e.ee.XY, where X and Y are values to be determined. Any distinction between **φ** and **z** is made entirely by AFL and AFR.

Notating foot-edges with parentheses, we observe that the original sequence FU has the structure (σσ)(σ). Only the left edges count for AFL, so the contribution of UF and FU to the overall AFL penalties is determined by the following reduced structures:

FU = (σ σ (σ *in* **φ** = […FU…]
UF = (σ (σ σ *in* **z** = […UF…].

These share whatever AFL penalty is induced by the leftmost foot-edge, but in **z** the second foot-edge is closer to the beginning of the word, thereby shaving one violation from its AFL score. Since everything else in forms **φ** and **z** is identical, we have AFL(**z**) < AFL(**φ**).

Applying the same reasoning to AFR, we have the following relevant representations:

FU = σ σ) σ) *in* **φ** = […FU…]
UF = σ) σ σ) *in* **z** = […UF…].

These share the AFR penalty induced by the second foot-edge, but in **φ** the first foot-edge is one syllable closer to the right edge of the word. Thus, AFR(**φ**) < AF(**z**).

Putting these results together, we obtain the ERC e.ee.WL. Since Pos.L gives us exactly the same ERC, we have trivially (since α⊨α) that Pos.L⊨**z**~**φ**. Therefore, in any grammar satisfying Pos.L, we have **z**≻**φ**, and **φ** cannot be optimal.

The argument with respect to Pos.R and UF proceeds identically, exchanging right and left. This establishes the lemma. ☐

(12) **Lemma**. No optimal output contains a unary foot displaced from the dominant edge.

Proof. Consider grammars satisfying Pos.L, and consider a form **φ** = […U…]. If the first "…" is nonempty, displacing the cited unary from the dominant (left) edge, then the cited U must sit in one of three configurations: oU, UU, FU. The first two of these are not present in optima by Lemmas (7) and (6) respectively. The third is absent from optima under Pos.L according to Lemma (11). If the cited U is displaced from the left edge, then **φ** is not a possible optimum.

The same argument may be replicated for Pos.R with mirror-image forms. ☐

(13) **Corollary**. **One U**. No optimal output contains more than one U.

Proof. Even if one U sits at the dominant edge, any other U must be displaced from the dominant edge, a guarantee of suboptimal status, by Lemma (12). ☐

(14) **Lemma**. No optimal output under Pos.L contains the sequence oF. No optimal output under Pos.R contains the sequence Fo.

Proof. Consider any grammar satisfying Pos.L. Let **φ** = […oF…]. Consider a competitor **z** in which the syllables parsed by the cited sequence oF in **φ** is parsed instead as Fo in **z**, so that we have **z** = […Fo…], where the competitors **φ** and **z** are identical in every respect outside the cited oF and Fo sequences. Since both **z** and **φ** consist of exactly the same units, they fare the same on Ps, Iamb, and Trochee and differ only on the alignment constraints.

We have the following AFL-relevant representations:

oF = σ (σ σ   *in* **φ** = […oF…]

Fo = (σ σ σ   *in* **z** = […Fo…].

The cited (left) foot-edge contributes one more violation to the score for AFL in **φ** than in **z**. Since everything else outside the cited sequences is the same in **z** and **φ**, we have AFL(**z**) < AFL(**φ**).

For evaluation by AFR, we have the following relevant representations:

oF = σ σ σ)   *in* **φ** = […oF…]

Fo = σ σ) σ   *in* **z** = […Fo…].

The cited (right) foot-edge contributes one violation less to the score for AFR in **φ** than in **z**. Since the two outputs are identical except for the cited sequences, it follows that AFR(**φ**) < AFR(**z**).

These lucubrations give us **z**~**φ** = e.ee.WL. Since this is the same as Pos.L, we have Pos.L⊨e.ee.WL, trivially. In any grammar satisfying Pos.L, we have **z**≻**φ**. It follows that **φ** cannot be optimal under Pos. L. This means that optimal outputs in Pos.L grammars cannot contain the sequence oF.

The same reasoning applies to grammars satisfying Pos.R, using mirror image forms. The result is that optima under Pos.R cannot contain Fo. □

(15) **Proposition**. **Positioning of Feet in Optima**. In any grammar satisfying Pos.L, any optimal output of length 2 or greater must be of the form $[(U)F^n o^k]$, $n \geq 1$, $k \geq 0$. In any grammar satisfying Pos.R, any optimal output of length 2 or greater must be of the form $[o^k F^n (U)]$.

Proof. Forms of length 2 are shaped [F] by Lemma (8), which guarantees the presence of at least one binary foot, meeting the claimed descriptions. Consider forms of length 3 or greater in grammars under Pos.L. If there is a unary foot in an optimal output, it cannot be displaced from the beginning, by Lemma (12). This means that only forms [(U)…], where "…" contains no U, have any hope of optimality. If the form has a binary foot, and it must have at least one by Lemma (8), then no such foot can appear in optima in the configuration oF, by Lemma (14). This leaves only the configuration $[(U)F^n$… for binary feet to occupy in optima. Completing the form with unparsed syllables runs afoul of no condition established so far, so $[(U)F^n o^k]$ is the only pattern left for optimal outputs to assume.

The same argument may be replicated for Pos.R mutatis mutandis. □

The optimal shapes established in Proposition (15) will be further refined as we proceed.

We conclude with a couple of remarks of more general interest. First, observe that *only* AFL and AFR are involved in positioning decisions in nGX. If two competing outputs have exactly the same number of feet of exactly the same type, they perform identically on the constraints Ps, Iamb, Troch, as noted in the arguments above. They can only be distinguished by foot location, which is monitored by AFL and AFR.

Second, and perhaps less obviously, in any contest between an optimum and a competitor that matches it in every characteristic except foot position, it is the dominant alignment constraint that decides between them. There are no cases where two such competitors, one optimal, both fare equally on the dominant constraint, leaving the decision to its subordinate antagonist. This is implicit in the proofs already given, and need merely be brought out.

(16) **Remark**. The dominant alignment constraint decides foot-location in competitions between an optimal form and a competitor with the same number and types of feet. The subordinate constraint makes no decisions.

Proof. By Proposition (15), all possible optima have either the shape $[(U)F^n o^k]$ or the shape $[o^k F^n (U)]$, $n \geq 1$, $k \geq 0$. We consider only forms of length 3 or greater. In forms of length 1, there is only one output admitted by GEN_{nGX}, so no competition. The same is true of length 2, because the only optima are shaped [F].

Consider the situation under Pos.L, and let **z** be an output possibly optimal under Pos.L, and therefore of the form $[(U)F^n o^k]$, $n \geq 1$, $k \geq 0$. Observe that **z** must contain at least one binary foot, by Proposition (8), motivating the requirement that $n \geq 1$. Let $\boldsymbol{\varphi}$ be a competing parse, $\boldsymbol{\varphi} \neq \mathbf{z}$, with exactly the same number and types of feet as **z**. We make no supposition as to whether $\boldsymbol{\varphi}$ is possibly optimal, although by assumption there is an arrangement of its feet that is, namely **z**.

The prosodic pattern of **z** is the only arrangement of its prosodic units that lacks configurations oU, Uo, oF, or FU. Therefore $\boldsymbol{\varphi}$ must contain at least one of these. Since $\boldsymbol{\varphi} \neq [Uo^k]$, if $\boldsymbol{\varphi}$ contains Uo, it must also contain either oU, oF, or FU; we may therefore set $[Uo^k]$ aside.

The proofs of Lemmas (11) and (14) establish that for any form $\boldsymbol{\varphi}$ containing a configuration FU or oF, respectively, we can produce another form *strictly* better than $\boldsymbol{\varphi}$ on AFL by virtue of lacking one of those configurations, namely by replacing them with UF and Fo respectively. The same reasoning that applies to oF in Lemma (14) extends to oU. Thus, for any form $\boldsymbol{\varphi}$ which differs from **z** only in disposition of feet we can construct a $\boldsymbol{\varphi}'$ that lacks one of the instances oU, oF, FU, with $AFL(\boldsymbol{\varphi}') < AFL(\boldsymbol{\varphi})$. But if $\boldsymbol{\varphi}'$ is itself not identical to **z,** then it must also contain at least one instance of oU, Uo, oF, or FU. Once again, any form containing Uo must also contain at least one of the others. We may therefore iterate the same reasoning until we reach a form to which it does not apply, by virtue of lacking oU, Uo, oF, FU. But this can only be **z**. Therefore $AFL(\mathbf{z}) < AFL(\boldsymbol{\varphi})$. This shows that the decision between the possible optimum **z** and any competitor with the same prosodic units differently disposed is made entirely by AFL in grammars under Pos.L.

The same reasoning applies mutatis mutandis to mirror-image forms under Pos.R. □

§3 MULT.sp/D: ⟨Adom, Fsub⟩<> Ps

The number of feet in optima is determined by Mult.D/sp. We introduce two useful descriptive terms, defined extensionally.

(17) **Sparse and dense**

a. A *sparse* language has exactly 1 foot in every output.
b. A *dense* language has more than 1 foot in *some* outputs forms.

These definitions suffice to distinguish the relevant classes of extensional languages in nGX. The terms are echoed in the values sp/D associated with the property Mult, but the value names refer to specific ranking relations, not to the characteristics of forms. The extensional content of the values Mult.sp and Mult.D will prove to be much more detailed than the broad distinction incorporated in the definitions of ‘sparse’ and ‘dense’ just given.

We begin the argument by noting that short optima have limited structural options.

(18) **Remark**. **Shrift for the Short**. Optimal outputs of length 1 are of the form [U]. Optimal outputs of the length 2 are of the form [F].

Proof. By Gen_{nGX} all outputs must contain at least one foot, settling the first case. In the case of length 2, observe that Lemma (6) rules out UU, and Lemma (7) rules [Uo] and [oU]. This leaves only [F]. ☐

§3.1 Mult.sp: ⟨Adom, Fsub⟩≫ Ps

We first consider the extensional effects of Mult.sp.

(19) **Lemma**. Under Mult.sp, no optimal output contains FF.

Proof. Let **z** = […Foo…] and **φ** = […FF…], where **z** and **φ** are identical in every respect outside the specified regions FF and Foo. We may also assume, without losing sight of any possible optima, that all binary F are of the dominant foot type, as Proposition (10) assures us, which we will assumed to be iambic by Ftyp.ia. We assume Adom = AFL by Pos.L. We have, then, Mult.sp = ⟨Troch, AFL⟩≫Ps, ERCwise L.eW.We. As always, we want to examine the ERC **z**~**φ**, so we determine the relative performance of the two competitors over the constraint set.

z = […Foo…], **φ** = […FF…]		
z~**φ**	Comparison of violation values	Rationale
L	Ps(**z**) > Ps(**φ**)	Form **z** has 2 more unparsed syllables than **φ**.
e	Iamb(**z**) = Iamb(**φ**)	All feet are identical in both, except for the cited sequences, which contain no non-iambic feet.
W	Troch(**z**) < Troch(**φ**)	Form **z** contains one fewer non-trochaic foot than **φ**.
W	AFL(**z**) < AFL(**φ**)	AFL sees Foo in **z** as (σ σ σ σ, and FF in **φ** as (σ σ (σ σ. In **φ**, the rightmost cited foot-edge incurs a penalty for AFL unmatched in **z**. The leftmost cited foot-edge in **φ** and in **z** incur the same AFL penalty.
e/W	AFR(**z**) ≤ AFR(**φ**)	AFR sees Foo in **z** as σ σ) σ σ and FF in **φ** as σ σ) σ σ). The leftmost cited foot-edge is identically placed in both. The rightmost foot-edge in **φ** incurs a positive penalty if FF is not final; if it is final in **φ**, it incurs no penalty.

Putting these considerations together, we have the ERC L.eW.We or L.eW.WW. Under FTyp.ia and Pos.L, Mult.sp = L.eW.We, which is identical to the first and asymmetrically entails the second by W-extension. Therefore, {FType.ia, Pos.L, Mult.sp}⊨**z**~**φ**, and we have **z**≻**φ** in grammars meeting these conditions. It follows that no form that like **φ** contains FF can be optimal in these grammars.

The argument replicates mutatis mutandis for the other values of FType.ia/tr and Pos.L/R, establishing the claim. □

(20) **Lemma**. Under Mult.sp, no optimal output contains UF or FU.

Proof. Assume for concreteness FType.ia and Pos.L. Lemma (12) guarantees that no optimum under Pos.L contains a unary foot displaced from the left edge, ensuring a fortiori that FU cannot appear in optima under the combination of Mult.sp and Pos.L. We therefore need only consider a form **φ** = [UF…] and contrast it with a form **z** = [Fo…], where the material in "…" is identical in the two forms.

z = [Fo…], **φ** = [UF…]		
z~φ	Comparison of violation values	Rationale
L	Ps(**z**) > Ps(**φ**)	Form **z** has one more unparsed syllable than **φ**.
W	Iamb(**z**) < Iamb(**φ**)	Form **φ** has one more non-iambic foot than **z**, namely the cited U, contributing 1 more violation of Iamb than **z** has.
W	Troch(**z**) < Troch(**φ**)	Form **φ** has one more non-trochaic foot than **z**.
W	AFL(**z**) < AFL(**φ**)	AFL sees Fo in **φ** as (σ σ σ and the sequentially corresponding syllables in UF in **z** as (σ (σ σ. The rightmost foot-edge in **z** incurs a penalty on AFL not matched in **φ**.
W	AFR(**z**) < AFR(**φ**)	AFR sees the UF structure in **φ** as σ) σ σ) and the Fo structure in **z** as σ σ) σ. The leftmost cited foot-edge in **φ** incurs a greater penalty on AFR than the *only* cited foot-edge in **z**.

Putting these observations together yields the ERC L.WW.WW which is clearly entailed by Mult.sp = L.eW.We. We have therefore {FType.ia, Pos.L, Mult.sp} ⊨**z**~**φ**. Thus **z**≻**φ**, and it follows that **φ** cannot be optimal in any grammar meeting these conditions.

The same reasoning applies mutatis mutandis when other values are chosen for FType.ia/tr and Pos.L/R, re-defining Adom and Fdom. In particular, analysis of mirror image forms under Pos.R shows that sparse optima cannot contain FU. □

(21) **Proposition**. **The shape of the sparse**. In grammars satisfying Mult.sp = ⟨Adom, Fsub⟩≫Ps, all optima of length 2 or greater take the form [Fo^k] under Pos.L and [$\mathrm{o}^k\mathrm{F}$] under Pos.R, $k \geq 0$.

Proof. Optima are restricted to the forms [$(\mathrm{U})\mathrm{F}^n\mathrm{o}^k$] and [$\mathrm{o}^k\mathrm{F}^n(\mathrm{U})$], $n \geq 1$, $k \geq 0$, under Proposition (15). By Lemma (19) no optimal output under Mult.sp contains FF, and by Lemma (20) no optimal output under Mult.sp contains UF or FU. Sparse optima can then only be of the form [Fo^k] and [$\mathrm{o}^k\mathrm{F}$]. □

§3.2 Mult.D: Ps ≫ ⟨Adom, Fsub⟩

From the property statement itself, we have Mult.D = Ps≫ ⟨Adom, Fsub⟩. Setting as always Fdom = Ia and Adom = AFL for ease of argument, we find ourselves dealing with grammars that satisfy the ERC W.eL.Le, which says Ps≫⟨AFL,Troch⟩. Fusing this with Pos.L = e.ee.WL, we have Pos.L◦Mult.D = W.eL.LL. We now proceed to deduce properties from this ERC, making use of the logical fact recorded as Remark (9) that if a coordinate value is L, we have nothing to verify about the corresponding coordinate in any ERC that we want it to entail. This means that we can content ourselves with establishing ERC fragments of a form that we can represent as W.e*x*.*xx*, where the *x*'s indicate unknown values, which of course needn't be the same.

(22) **Lemma.** Under Mult.D, no optimal output contains oo.

Proof. Let **φ** = […oo…]. We compare with the minimally differing **z** = […F…], where F is of the dominant foot type, and where the sequentially corresponding "…" are structurally identical in **z** and **φ**. We assume for concreteness FTyp.ia, so that F is iambic in optima.

z = […F…], **φ** = […oo…]		
z~φ	Comparison of violation values	Rationale
W	Ps(**z**) < Ps(**φ**)	Form **z** has 2 fewer unparsed syllables than **φ**.
e	Iamb(**z**) = Iamb(**φ**)	Neither F nor oo contributes violations of Fdom = Iamb.

Of the ERC **z~φ**, we now know the crucial values. From **z~φ** = W.*ex.xx* and Pos.L◦Mult.D = W.eL.LL, we may conclude, in light of Remark (9), that

Pos.L◦Mult.D⊨**z~φ**

and therefore that **z**≻**φ**. The argument reproduces mutatis mutandis for all other settings of the Pos.L/R and FTyp.ia/tr values. It follows that no output […oo…] may be optimal under Mult.D. □

(23) **Lemma**. Under Mult.D, no optimal output contains both U and o.

Proof. With Fdom = Iamb and Adom = AFL, any optimal output must be of the form [(U)$F^n o^k$], with all F iambic, by Proposition (10) and Proposition (15). From Lemma (22), we know that $k \leq 1$, since dense outputs cannot contain oo. Therefore, any optimal output under Mult.D must be of the form [(U)F^n(o)]. Consider a U-containing form **φ** that contains both U and o. We must have **φ** = [UF^no]. This form cannot be of odd length, so we conclude that no odd length form may contain both U and o.

Now consider the competitor **z** = [F^{n+1}]. We have these relations on Ps and Iamb.

z = [F^{n+1}], **φ** = [UF^no]		
z~φ	Comparison of violation values	Rationale
W	Ps(**z**) < Ps(**φ**)	Form **φ** contains an unparsed syllable while **z** does not.
W	Iamb(**z**) < Iamb(**φ**)	Form **φ** contains the non-iambic foot U while **z** does not.

This gives us as much of the ERC **z~φ** as we need. We have **z~φ** = W.W*x.xx*. Since Pos.L∘Mult.D = W.eL.LL, we have

Pos.L∘Mult.D⊨**z~φ**

This establishes that **z**≻**φ**, and that therefore **φ** cannot be optimal. No form under Pos.L, FTyp.ia, and Mult.D may contain both U and o.

As always, these arguments may be replicated under all choices of values for FType.ia/tr and Pos.L/R. It follows that no optimal outputs of any length, odd or even, may contain both U and o under Mult.D. ☐

(24) **Proposition**. **The shape of the dense**. In grammars satisfying Mult.D = Ps ≫ ⟨Adom,Fsub⟩, all optimal outputs of length 2 or greater take the form [F^n(o)] or [(U)F^n] under Pos.L and [(o)F^n] or [F^n(U)] under Pos.R, $n \geq 1$.

Proof. Optima are restricted by Proposition (15) to the forms [(U)$F^n o^k$] and [$o^k F^n$(U)]. Under Mult.D, optimal outputs may not contain oo by Lemma (22), nor may they contain both U and o by Lemma (23), so under Pos.L the only dense optima have the shapes [F^n(o)], [UF^n], and under Pos.R, the possible optima must be shaped as their mirror images. ☐

(25) **Corollary**. All dense even-length outputs are exhaustively parsed into binary feet.

Proof. Forms [F^no] and [UF^n] are of odd length. This leaves only [F^n] among the patterns of optima admitted by Proposition (24). ☐

§4 Un.o/X: ⟨Adom, Fdom⟩<> Ps

All that remains to be determined is whether a dense odd-length output has an unparsed syllable or a unary foot. The property is determined by Un.o/X: ⟨Adom, Fdom⟩ <> Ps.

(26) **Lemma**. **Un.o/X and Sparseness**. Sparse grammars cannot assume the value Un.X. Sparse grammars must assume the value Un.o.

Proof.We show that the values Un.X and Mult.sp are contradictory.

[1] Mult.sp = ⟨Adom, Fsub⟩ ≫ Ps.

[2] Fdom ≫ Fsub.

[3] Mult.sp⊨⟨Adom, Fdom⟩≫Ps = Un.o, from [1],[2] by transitivity of domination.

[4] Un.X = Ps ≫ ⟨Adom, Fdom⟩.

[5] {Mult.sp,Un.X}⊨⟨Adom,Fdom⟩≫ Ps; and Ps≫ ⟨Adom,Fdom⟩, from [3], [4].

Recall that "⟨A,B⟩≫C" means "(A≫C) *or* (B≫ C)", while "C≫⟨A,B⟩" is "(C≫A) *and* (C≫B)." Domination is a strict order, and therefore asymmetric, so [5] is a contradiction. This leaves only the value Un.o = ⟨Adom, Fdom⟩≫ Ps to be available for conjunction with Mult.sp. Un.o is not only consistent with Mult.sp = ⟨Adom, Fsub⟩≫Ps, but is logically entailed by it, by [3]. □

The choice between the values of Un.o/X is therefore only relevant to the grammars of the dense languages.

(27) **Lemma**. Under Un.X, no optimal output contains o.

Proof. Un.X = Ps≫⟨Adom, Fdom⟩. Since Adom≫Asub and Fdom≫Fsub, Un.X delivers the ERC W.LL.LL no matter which values of Pos.L/R and FTyp.ia/tr are chosen. Let **z** be any form that is fully parsed, and **φ** a competitor with at least one unparsed syllable. Clearly **z**~**φ** = W.*xx*.*xx*. By the logic of Remark (9), this is entailed by W.LL.LL no matter what values are assigned to the individual *x*'s, so under Un.X we have **z**≻**φ**, eliminating **φ**'s chances for optimality, and with it the chances of any form with unparsed syllables. □

(28) **Proposition. The shape of Un.X.** In grammars satisfying Un.X = Ps ≫ ⟨Fdom, Adom⟩, all optimal outputs of length 2 or greater take the form [(U)F^n] under Pos.L and [F^n(U)] under Pos.R, $n \geq 1$.

Proof. From Lemma (26), no Mult.sp grammar can assume the value Un.X, so all Un.X grammars can only be Mult.D. Indeed, Un.X = Ps ≫ ⟨Adom, Fdom⟩ entails Mult.D = Ps≫⟨Adom, Fsub⟩ because Fdom ≫ Fsub. From Proposition (24), under Mult.D, optima are of the form [F^n(o)] or [(U)F^n] assuming Pos.L, and the mirror images under Pos.R. From Lemma (27), in all grammars satisfying Un.X, optimal outputs are completely parsed, eliminating outputs of the form [F^no] and [oF^n]. This leaves only [(U)F^n] under Pos.L and [F^n(U)] under Pos.R. □

(29) **Lemma**. Under Un.o, no optimal output of length 2 or greater contains U.

Proof. Un.o = ⟨Adom, Fdom⟩≫Ps. From Lemma (26)[3], Mult.sp ⊨ Un.o. Grammars under Mult.sp allow no U in optimal outputs of length greater than 1 syllable by Proposition (21), so this class of grammars under Un.o satisfies the statement of the lemma. We turn now to the remaining class of grammars, those satisfying Mult.D. Consider any dense form **φ** = [UF^n] and a dense competitor **z** = [F^no]. Setting Adom = AFL, Fdom = Iamb, we have the following ERC from fusion: Mult.D∘Un.o∘FTyp.ia∘Pos.L = L.WL.LL.
Explicitly,

Mult.D	`W.eL.Le`
Un.o	`L.We.We`
FTyp.ia	`e.WL.ee`
Pos.L	`e.ee.WL`
fuse(all)	`L.WL.LL`

By Remark (9), we need only show that Fdom (Iamb) assigns fewer violations to **z**, thus ensuring a W in the second coordinate of the ERC **z**~**φ**. Forms **φ** = [UF^n] and **z** = [F^no] have the same number of binary feet, which we may assume to be of the dominant, iambic type. But **φ** contains an additional unary foot, adding one to its Iamb penalty. This gives the ERC *x*.W*x*.*xx*, which is entailed by Mult.D∘Un.o∘FType∘Pos.L = L.WL.LL. In any such grammar, **z**≻**φ**. Parallel arguments hold by switching the values of Fdom and Adom. Switching Adom does not affect the fused ERC. Switching Fdom gives us the ERC L.LW.LL, and the same form of argument goes through under the assumption that F in the competitors is trochaic. □

(30) **Proposition. The shape of Un.o.** In grammars satisfying Un.o, optima are of the form [$F^n o^k$] under Pos.L and [$o^k F^n$] under Pos.R, $n \geq 1$, $k \geq 0$.

Proof. Consider first grammars satisfying Mult.sp. By Lemma (26)[3] these must have the value Un.o. From Proposition (21), we have that in all such grammars optima are of the form [Fo^k] and [$o^k F$], which accords with the claim that Un.o grammars admit only [$F^n o^k$] and [$o^k F^n$] under Pos.L and Pos.R respectively. Now consider grammars satisfying Mult.D. By Proposition (24), all optima in Mult.D grammars are of the form [(U)F^n] and [F^n(o)] under Pos.L and the mirror images under Pos.R. But by Lemma (29), no optimal outputs of length 2 or greater contain U under Un.o. Therefore only only [F^n(o)] and [(o)F^n] remain. These are of the form demanded by the lemma. □

Mootness. Not all properties impose distinctions on every class of grammars. For example, the property Un.o/X distinguishes one class of dense languages from another, but makes no distinction between types of sparse languages.

If Un.o/X = ⟨Fdom, Adom⟩<> Ps, as assumed above, then Un.X contradicts Mult.sp and Un.o is entailed by Mult.sp, as shown in Lemma (26). In this case, the ineffectiveness of Un.o/X follows from the logic of its formulation.

We may also give the property a simpler formulation as Fdom <> Ps, omitting mention of Adom, if we explicitly limit its scope to the grammars satisfying Mult.D. Under this approach, Alber & Prince describe the property Un.o/X as being *moot* with respect to Mult.sp. Nothing about the arguments above would change if we shifted to the scope-limited version. In nGX, there is no particularly strong reason to choose one over the other, but Alber & Prince show that mootness is a fundamental, ineradicable characteristic of typological structure in the general case which arises because grammars, understood as sets of rankings, are delimited by ERC sets.

§5 Extensional Characteristics of the optima of nGX

The results of §§1-4 classify the languages of nGX based on the property values that define their grammars. The property Un.o is given in its wide-scope version.

(31) nGX by Properties

Optima	Properties			
	Mult	FType	Pos	Un
Fo^k	sp	ia/tr	L	o
$o^k F$	sp	ia/tr	R	o
F^n(o)	D	ia/tr	L	o
(o)F^n	D	ia/tr	R	o
(U)F^n	D	ia/tr	L	X
F^n(U)	D	ia/tr	R	X

We now justify the classification.

(32) **Theorem**. **Optimal Outputs of nGX**. The optimal outputs of nGX of length greater than or equal to 2 syllables are drawn from the following patterns. Each, by GEN_{nGX}, is a single PrWd. Within each schema, F is uniformly iambic or uniformly trochaic, and $n \geq 1$, $k \geq 0$.

Mult/Un values	Pos.L	Pos.R
Mult.sp, Un.o	Fo^k	o^kF
Mult.D, Un.o	F^n(o)	(o)F^n
Mult.D, Un.X	(U)F^n	F^n(U)

Proof. From Proposition (15) we have that in any grammar satisfying Pos.L, optimal outputs take the form [(U)$F^n o^k$] and in any grammar satisfying Pos.R, optimal outputs take the form [$o^k F^n$(U)]. By Proposition (10), the binary feet in such forms are either all iambic or all trochaic. According to Proposition (21), in any grammar satisfying Mult.sp, optimal outputs must be of the form [Fo^k] or [o^kF] with a single binary foot per word. From Proposition (24), in any grammar satisfying Mult.D, optimal outputs are restricted to the form [F^n(o)] or [(U)F^n] under Pos.L and [(o)F^n] or [F^n(U)] under Pos.R, $n \geq 1$. A grammar with the value Un.X has only completely parsed forms [(U)F^n] or [F^n(U)] by Proposition (28), and in any grammar with the value Un.o, there are no optima with unary feet, by Proposition (30), allowing only the forms [F^n(o)] or [(o)F^n]. □

6. Universal Supports for nGX

(33) **Theorem**. **Universal Supports for nGX**. Any collection of candidate sets that delivers the 12 grammars obtained from the properties in (4) is a universal support for nGX.

Proof. The properties in (4) entail the characterization of the optima asserted in Theorem (32). Each extensional description, when variables *n* and *k* are specified and when any parenthesized element is either included or omitted, determines the length of the described form in syllables, the location of its feet with respect to the syllable string and, when F is specified as iambic or trochaic, the location of the heads of the feet. By Remark (2), there can be only one form admitted by GEN_{nGX} that meets this description. Therefore, no further refinements of the grammars can be motivated. □

We may also now completely characterize the universal supports of nGX. The key observation is that a universal support must distinguish all the possible optima. The simplest case is when the optima manifest in a single candidate set.

(34) **Lemma. The Long Supports**. Any odd-length input of 5 or more syllables provides a universal support for nGX.

Proof. Consider an input of length $2m+1$, $m \geq 2$. Since it admits all parses defined by GEN_{nGX}, it admits exactly these from the table accompanying Theorem (32).

Mult/Un values	Pos.L	Pos.R
Mult.sp, (Un.o)	Fo^{2m-1}	$\mathrm{o}^{2m-1}\mathrm{F}$
Mult.D, Un.o	$\mathrm{F}^{m}\,\mathrm{o}$	$\mathrm{o}\,\mathrm{F}^{m}$
Mult.D, Un.X	$\mathrm{U}\,\mathrm{F}^{m}$	$\mathrm{F}^{m}\,\mathrm{U}$

Split each cell in two by allowing F to be uniformly iambic or uniformly trochaic in the formulae. These 12 forms are exactly the optimal outputs admitted by each of the grammars of nGX, as we have shown. Thus the candidate set for input of length 2*m*+1, *m*≥2, contains 12 distinct optima, one for each grammar of nGX. □

(35) **Lemma**. **Failure of the Even**. No even length provides a universal support for nGX.

Proof. The input of length 2 yields only the two optima shaped [F], by Remark (18). For the lengths 2*m*, *m*≥2, we turn to Theorem (32), which gives a complete list of the shapes of all possible optima of nGX. No forms [F^no], [UF^n] [oF^n], [F^nU] can appear as parses of even lengths. The possibly optimal parses of even length 4 or greater are exhaustively those of the shapes [Fo^{2k}], [o^{2k}F], and [F^n], *n*≥2, *k*≥0. Among the dense, which fall under Mult.D, the longer even length inputs fail therefore to distinguish between Un.o and Un.X. □

We know from experiment that the support with inputs of length 3 and 4 syllables is universal, simply because it produces the 12 grammars. We may now generalize this observation to cover the collocation of the length 3 cset with any other cset with input of even length greater than 2.

(36) **Lemma**. The support with two inputs, one of length 3, the other of length 2*m*, *m*≥2, is universal and minimal.

Proof. Length 3 has the optima [Fo], [oF], [UF], [FU], for F iambic and for F trochaic. Observe that the optimal structures [Fo^k] and [F^n(o)] are not distinguished at length 3, nor are [o^kF] and [(o)F^n]. This establishes that the input of length 3 does not provide a universal support.

The structural types neutralized at length 3 are however distinguished at length 2*m*, *m*≥2, where the optima [$\mathrm{F\,o}^{2m-2}$] instantiate [Fo^k], contrasting with the optima [F^m] of the shape [F^n(o)]; and the same mirror-imagewise for [$\mathrm{o}^{2m-2}\,\mathrm{F}$]. From Corollary (35), we know that the optima of length 2*m* are [Fo^{2k}], [o^{2k}F], for *k* = *m*–1, and [F^m], doubled by the iambic-trochaic distinction. From both lengths taken together, we may form pairs that distinguish all the grammars. Under Pos.L, these take the schematic shapes (Fo, Fo^{2k}), (Fo, F^m), and (UF, F^m), *k* = *m*–1, which may be expanded to instantiate the full range of Pos.L/R and FTyp.ia/tr values, yielding the 12 grammars. This establishes universality. Since neither length 3 nor length 2*m*, *m*≥2, is universal by itself, we have minimality as well. □

(37) **Theorem**. **Minimal Universal Supports for nGX**. The minimal universal supports for nGX are (1) the csets from an input of length 3 and an input of length 2*m*, *m*≥2**,** and (2) any single cset from an input of odd length 5 syllables or greater. There are no others.

Proof. Lemma (36) establishes that the combination of csets from lengths 3 and $2m$, $m \geq 2$ is universal and minimal. Lemma (34) establishes the odd lengths greater than or equal to 5 syllables are each universal, with minimality following trivially. No even length can be universal by itself, by Lemma (35).

Any support Σ that contains an even length cset must therefore contain another cset of odd length. If the length of the odd member is greater than or equal to 5, then Σ is non-minimal. We now dispose of the shorter lengths. The 1 syllable cset has the same optimum in all grammars, and therefore belongs to no minimal support.

The 2-syllable input has only two optima, both shaped F, for iambic and trochaic. Every optimal output of length greater than 1 contains F, by Proposition (8). That F must be either iambic or trochaic, so that every such output determines the value of FTyp.ia/tr. The 2-syllable cset may therefore be removed from any universal support without compromising universality. We conclude that the 1 and 2 syllable csets belong to no minimal universal supports, and with that, we have covered all the cases. □

Along the route to characterizing the universal supports for nGX, we have also established the shape of every optimum admitted by the system, and associated each optimum with the grammar that admits it. The essential move was to connect each grammar-defining property value with the extensional characteristics that it imposes. The resulting view of the system now lies well beyond what can be abstracted from observing the distribution of characteristics in a finite sample from nGX. We definitively *have* the typology in a way that sets the stage for wide-ranging analysis and comparison with other systems, abstract and concrete.

■

Appendix 1

Sample of Extensional typology of nGX: 2σ-5σ inputs. NB. All 1σ inputs are U = -X-.

Language	2s	3s	4s	5s	Footing
sp.ia.L(.o)	-uX-	-uX-o-	-uX-o-o-	-uX-o-o-o-	Fo^n
sp.tr.L(.o)	-Xu-	-Xu-o-	-Xu-o-o-	-Xu-o-o-o-	
sp.ia.R(.o)	-uX-	-o-uX-	-o-o-uX-	-o-o-o-uX-	o^nF
sp.tr.R(.o)	-Xu-	-o-Xu-	-o-o-Xu-	-o-o-o-Xu-	
D.ia.L.o	-uX-	-uX-o-	-uX-uX-	-uX-uX-o-	F^n(o)
D.tr.L.o	-Xu-	-Xu-o-	-Xu-Xu-	-Xu-Xu-o-	
D.ia.R.o	-uX-	-o-uX-	-uX-uX-	-o-uX-uX-	(o)F^n
D.tr.R.o	-Xu-	-o-Xu-	-Xu-Xu-	-o-Xu-Xu-	
D.ia.L.X	-uX-	-X-uX-	-uX-uX-	-X-uX-uX-	(U)F^n
D.tr.L.X	-Xu-	-X-Xu-	-Xu-Xu-	-X-Xu-Xu-	
D.ia.R.X	-uX-	-uX-X-	-uX-uX-	-uX-uX-X-	F^n(U)
D.tr.R.X	-Xu-	-Xu-X-	-Xu-Xu-	-Xu-Xu-X-	

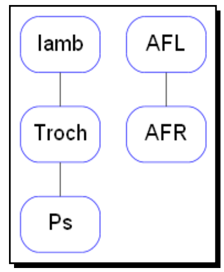
Appendix 2

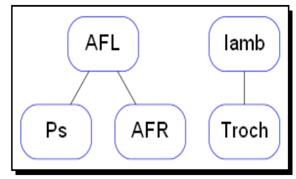
Grammars of nGX given as SKBs and Hasse diagrams, from OTWorkplace.

Appendix 2.1 'Sparse'. Outputs = [F,o^n]

sp.ia.L

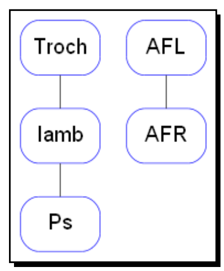
4:AFL	5:AFR	2:Iamb	3:Troch	1:Ps
W	**L**			
		W	**L**	
W			**W**	**L**

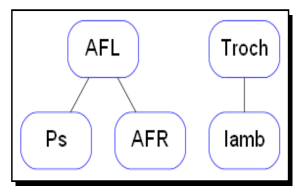




sp.tr.L

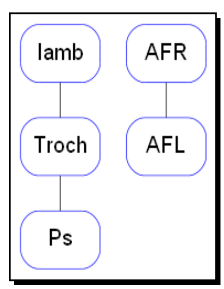
4:AFL	5:AFR	3:Troch	2:Iamb	1:Ps
W	**L**			
		W	**L**	
W			**W**	**L**

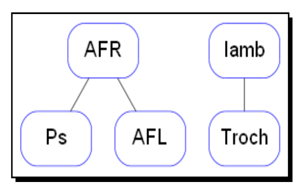




sp.ia.R

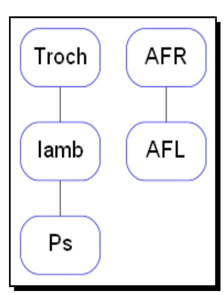
5:AFR	4:AFL	2:Iamb	3:Troch	1:Ps
W	**L**			
		W	**L**	
W			**W**	**L**

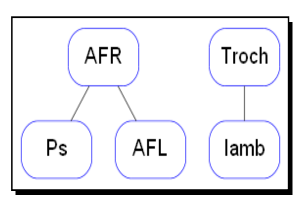




sp.tr.R

5:AFR	4:AFL	3:Troch	2:Iamb	1:Ps
W	**L**			
		W	**L**	
W			**W**	**L**

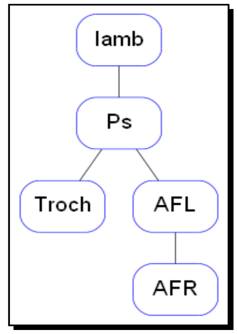




Appendix 2.2 D.o = 'Weakly Dense'. Outputs = $[F^n,o]$

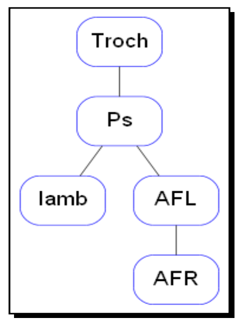
D.ia.L.o

2:Iamb	1:Ps	3:Troch	4:AFL	5:AFR
W	**L**			
	W	**L**	**L**	
			W	**L**



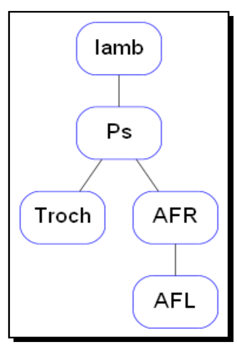
D.tr.L.o

3:Troch	1:Ps	2:Iamb	4:AFL	5:AFR
W	**L**			
	W	**L**	**L**	
			W	**L**



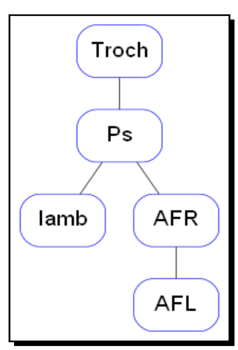
D.ia.R.o

2:Iamb	1:Ps	3:Troch	5:AFR	4:AFL
W	**L**			
	W	**L**	**L**	
			W	**L**



D.tr.R.o

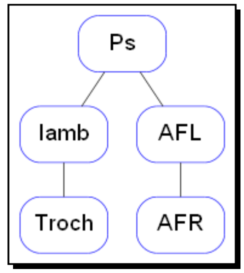
3:Troch	1:Ps	2:Iamb	5:AFR	4:AFL
W	**L**			
	W	**L**	**L**	
			W	**L**



Appendix 2.3 D.X = ‘Strongly Dense’. Outputs = [F^n,U]

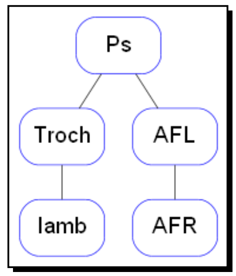
D.ia.L.X

1:Ps	2:Iamb	4:AFL	3:Troch	5:AFR
W	**L**	**L**		
	W		**L**	
		W		**L**



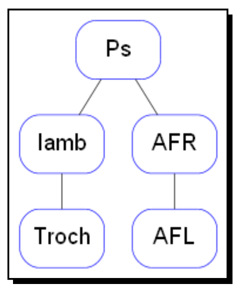
D.tr.L.X

1:Ps	3:Troch	4:AFL	2:Iamb	5:AFR
W	**L**	**L**		
	W		**L**	
		W		**L**



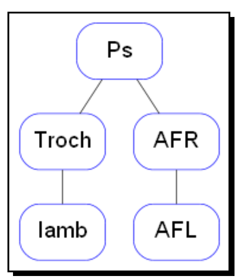
D.ia.R.X

1:Ps	2:Iamb	5:AFR	3:Troch	4:AFL
W	**L**	**L**		
	W		**L**	
		W		**L**



D.tr.R.X

1:Ps	3:Troch	5:AFR	2:Iamb	4:AFL
W	**L**	**L**		
	W		**L**	
		W		**L**



Appendix 3 Property specifications in the Iambic Left Quadrant of nGX

(i) sp.ia.L.o "Sparse Iambic Left"

Property	Value	ERC
Mult	sp	`L.eW.We`
Ftyp	ia	`e.WL.ee`
Pos	L	`e.ee.WL`
Un	o	`L.We.We`

(ii) D.ia.L.o "Weakly Dense Iambic Left"

Property	Value	ERC
Mult	D	`W.eL.Le`
Ftyp	ia	`e.WL.ee`
Pos	L	`e.ee.WL`
Un	o	`L.We.We`

(ii) D.ia.L.X "Strongly Dense Iambic Left"

Property	Value	ERC
Mult	D	`W.eL.Le`
Ftyp	ia	`e.WL.ee`
Pos	L	`e.ee.WL`
Un	X	`W.Le.Le`

7. References

ROA = Rutgers Optimality Archive (roa.rutgers.edu)

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