From Intensional Properties to Universal Support

Birgit Alber, Natalie DelBusso, and Alan Prince
University of Verona and Rutgers University

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ABSTRACT

An OT system is specified by defining its constraints and the structures they evaluate. These give rise to a set of grammars, the *typology* of the system, which emerges from the often complex interactions among constraints and structures. Every typology is determined by a finite collection of candidate sets (csets). How do we know that we have assembled *universal support*, a collection of csets sufficient to distinguish all grammars of the system? Lacking a universal support, we do not have the typology and we cannot deal systematically with its structure and consequences.

This concrete question can be answered in terms of an enhanced abstract understanding of typological structure. Under Property Theory (Alber & Prince 2015, in prep.), a typology is resolved into a set of *properties,* ranking conditions that have mutually exclusive *values*. When the structural correlates of each value are determined, the ranking values defining a grammar also determine the extensional *traits* exhibited in its optima. Suppose we have the property analysis of a typology derived from a proposed support for an OT system. If every consistent choice of values ensures that a *single* optimum is chosen in every cset admitted by the system, then no grammar derived from the proposed support can be split by consideration of further csets, and that support must be universal for the system. This method of proof is applicable to any OT system. Here we use it to analyze the prosodic system nGX (Alber & Prince in prep.), determining its universal supports and the shape of the forms made optimal by its grammars.

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TABLE OF CONTENTS

0. INTRODUCTION

A *factorial typology* is a set of grammars. We are not given the grammars directly, but must deduce them from the way that the posited constraints deal with the posited structures. How do we know that the candidate sets we have examined are sufficient to discriminate all the grammars that are allowed by our assumptions? This is the problem of finding a *universal support* for a typology. Without a universal support, we don't have the typology, and without the typology, many types of systematic claims about it must languish unjustified.

Here we show how the universal status of a proposed support may be established when we have exact descriptions of the types of optima allowed in the grammars. If a typology is factored into (intensional) ranking *properties* in the sense of Alber & Prince (in prep.; henceforth A&P), and if the property values are associated with (extensional) *traits* carried by optima, then a grammar as a combination of values is associated with a description of its optima as a conjunction of the structural traits associated with the ranking values. If the descriptions thereby obtained uniquely denote single candidates, then the grammars cannot be further refined, and the support that produced the grammars must be universal.

This method of associating extensional traits with ranking patterns answers a much more general question: *what do the languages of the typology look like*? Since a typology is generated from a finite sample of candidate sets, we cannot in general be satisfied with remarking about the distribution of traits in the sample. We must use the grammars to project over the entire set of optima. The grammatical structure relevant to this enterprise is encoded in the ranking properties that combine to give the grammars.

To set the stage, we provide a concise account of the basic notions and terminology. An OT system S is defined by specification of G_{EN_S} and C_{ON_S}. G_{EN_S} defines the structure of the candidates admitted by S and delimits how they are organized into the *candidate sets* (csets) in which competition takes place. In the most familiar case, a candidate is an input-output pair and each cset is derived from a single input. C_{ON_S} defines each constraint of S as a function from candidates to the nonnegative integers, which are interpreted as penalties and termed *violations*, providing the basis for determining optimality. A *language* is the collection of optimal candidates from every cset admitted by G_{EN_S}.

Optimality is defined in 3 steps. [1] A candidate *is better on a constraint than* a competing candidate if the constraint assigns it fewer violations than its competitor. [2] Given a linear order or *ranking* of all the constraints in C_{ON_S}, a candidate is *better on that ranking* than a competitor if it is better on the highest-ranked constraint that assigns them different violation values. [3] A candidate is *optimal* in its cset with respect to a given ranking if no other candidate in that cset is better on that ranking; an optimum is thus better than all other competitors that are distinct from it in constraint-assessed violations.

The *typology* of S is the collection of all grammars admitted by the definition of S. An individual (*ranking*) *grammar* is a non-empty set of linear orders on CON_S that all produce the same collection of optima, the same *language*. Every ranking grammar is characterizable as an

ERC set; [1] the rankings that belong to it are termed the ***l***inear ***e***xtensions of the ***g***rammar's ERC set, acronymically its 'legs'. It follows from the definition of the theory that a typology is always determined by a finite collection of candidate sets (csets), even when an infinite number of csets is admitted by G_{EN_S}.

A *support* for a single grammar is a collection of csets that suffices to delimit all the legs of the grammar when the appropriate candidates are selected as optima. A *universal support* for a typology is a collection of csets that suffices to deliver all of its grammars when every possible combination of optima is considered. A *minimal* universal support is a universal support from which no cset can be removed without destroying its universality. These concepts and terminology, which develop and further specify those of Prince & Smolensky 1993/2004 (henceforth P&S), are from Prince 2014, 2014-15, 2015, Merchant & Prince (in prep.), and A&P (2015, in prep).

The logic of OT analysis requires not only that the system S under scrutiny be defined by spelling out $\langle \text{GEN}_S, \text{CON}_S \rangle$, but that the typology claimed for S be derived from a valid universal support. As long as each cset contains all of its possible optima, the source of failure will be omission of at least one necessary cset. An insufficient support yields a typology *coarser* than the real one. This means that while the distinctions between grammars that are established by the support are veridical, there are yet further distinctions that must be made to arrive at the real typology, which is *finer* than the generated one. Some of the grammars of the coarser typology will turn out to be amalgams of two or more grammars from the actual typology of the system S. In particular, some ranking relations that are left open in grammars of the coarser typology must be specified, splitting some of the grammars, in order to arrive at the correct typology.

An instructive example is provided by the Basic Syllable Theory (BST) of P&S:ch. 6.[2] These authors define $\langle \text{GEN}_{BST}, \text{CON}_{BST} \rangle$ and go on to analyze 9 grammars of the BST Typology. As Riggle (2004) was the first to announce, the actual number of grammars is 12. The shortfall is due to the omission of an input that contains a consonant, which, under the limitations of Gen_{BST}, *cannot be faithfully syllabified in any optimal form*. P&S consider two csets: one from /V/, the other from /CVC/. The first gives rise to the triple fates of vowels that cannot be faithfully syllabified into syllables shaped $[_\sigma CV]$:

a. $V \rightarrow [_\sigma V]$	faithful reproduction as onsetless syllable
b. $V \rightarrow [_\sigma CV]$	epenthesis of C into onset
c. $V \rightarrow \varepsilon$	deletion of the vowel.

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[1] ERC = E(lementary) R(anking) C(ondition), Prince 2002a,b, *et seq*. Interpreting constraints as comparators of competing pairs of candidates, each constraint assigns W to a pair in which the first is better than the second, L to a pair in which the second is better than the first, and *e* to a pair in which neither is better than the other. The rankings in which the first is better than the second over an entire ranking are those in which *every L is dominated by some W*. (If the ERC contains both W and L, this is equivalent to *some W dominates every L*.) This is the 'Elementary Ranking Condition' proper. The term is extended to refer to the lists of values W,L,*e* that are so interpreted. For any given cset, the set of rankings that simultaneously satisfy all the ERCs derived from placing a certain candidate in first position and all others in second position is exactly the set of rankings under which that candidate is optimal.

[2] See also Prince 2015.

The second input gives rise to the triple fate of consonants that cannot be faithfully syllabified into [$_\sigma$CV] syllables, but *which can be faithfully syllabified* into licit syllables [$_\sigma$CVC]:

a. CVC → [$_\sigma$CVC] faithful reproduction as a closed syllable
b. CVC → [$_\sigma$CV][$_\sigma$CV] epenthesis of V
c. CVC → [$_\sigma$CV] deletion of C.

The mappings from each set cross-combine freely to yield grammars, and $3\times3 = 9$.

Missing is an input like /C/, for which there is no faithful parse. Under GEN$_{BST}$, all syllables contain a V, disallowing the structure [$_\sigma$C]. The two optimal outputs from /C/ are both unfaithful:

a. C → [$_\sigma$CV] epenthesis of a vowel to support the input C
b. C → ε deletion of C.

The empty string 'ε' lacks syllables altogether and therefore slips past all of their defining requirements. Both optimal outputs from /C/ satisfy all markedness constraints, so that the choice between them is made purely on grounds of faithfulness. The ranking f.max≫f.depV yields the epenthetic optimum, and f.depV≫f.max yields the deletional, empty output.[3] It follows that *every* grammar in the BST typology crucially ranks these two faithfulness constraints with respect to each other, because an OT grammar finds an optimum in every cset. In the BST, every grammar must contend with the cset derived from /C/ and along with it all those csets derived from any input containing a C that cannot be faithfully syllabified under GEN$_{BST}$.

Augmenting the support to universality therefore splits those grammars of the coarser P&S typology in which f.depV and f.max are freely ranked with respect to each other: those in which /CVC/ is faithfully reproduced as [$_\sigma$CVC]. Each such coda-allowing grammar is composed of legs taking the form of λ_1 and λ_2 below, where the material in the sequentially corresponding sequences "…" is assumed identical across the legs.

λ_1 = …f.max … ≫ …f.depV… ≫ …m.NoCoda…
λ_2 = …f.depV… ≫ …f.max … ≫ …m.NoCoda…

These legs produce the same optimum in the candidate set from the input /CVC/. The candidate CVC→[$_\sigma$CVC] is optimal on both legs, because in both the relevant faithfulness constraints dominate the relevant markedness constraint. Since there is neither epenthesis nor deletion in the optimum, no decision can be made between them. For the input /C/, however, the legs produce different optimal outputs, with λ_1 sponsoring C→[$_\sigma$CV] by virtue of f.max≫f.dep and λ_2 sponsoring C→ε by virtue of f.dep≫f.max. This means that legs of the form λ_1 and λ_2 cannot belong to the same grammar in BST.

From the BST point of view, the coda-allowing grammars derived from the support based on /V/ and /CVC/ are not grammars at all but mere collections of rankings. They lack the key grammar-defining property that we can call *uniform selection*, whereby all linear orders belonging to a grammar select the same optima — the same extensional language. Failure of uniform selection marks a proposed support as non-universal.

[3] The distinction between markedness and faithfulness constraints is fundamental to the way the BST works. We follow modern best practices by explicitly 'typing' each constraint for its class with the prefix 'm' or 'f'.

Uniform selection also gives us a way to establish the universality of a proposed support. Suppose we can show that each grammar derived from a proposed support selects a *single* violation profile as optimal in every candidate set admitted by $\text{G}\textsc{en}_{\text{S}}$. Then no grammar submits to further refinement: the support must be universal for S. On the face of it, this might seem like a burdensome requirement, inviting piecemeal study of all grammars and all candidate sets, but it takes on a different character in the context of the Classification Program of A&P.

A&P analyze the ranking structure of the grammars of a typology into a set of (intensional) *properties*. Each property takes on a set of mutually exclusive *values*, where each value is a ranking condition. The grammars of the typology, understood as ERC sets that delimit sets of rankings as their legs, are generated in their entirety by selecting values from the properties, with the possibility of scope restrictions that limit the freedom of combination. In the stress system nGX, examined in detail below, the property FTyp.ia/tr requires a choice between Iamb≫Trochee and Trochee≫Iamb. Every grammar must choose one or the other of these, and every leg of a grammar must accord with the choice. Similarly, the property Pos.L/R requires a choice between AFL≫AFR and AFR≫AFL, where AFL abbreviates the Generalized Alignment constraint commonly called ALL-FEET-LEFT and similarly for AFR, with exchange of chirality. A&P show that choosing values from these plus two other additional properties generates the entire set of ERC grammars obtained from their support for the nGX typology.

These *properties* are intensional in that they determine grammars, but they have extensional consequences, determining the structural *traits* that appear in optima, such as the shape and distribution of prosodic units or the appearance of epenthetic elements. For example, as we will show below, in optimal outputs of grammars that satisfy the nGX property value FTyp.ia, *every* binary foot in every word is iambic; in those that satisfy FTyp.tr *every* binary foot is trochaic. This is the extensional force of the intensional property FTyp in the system nGX.

It is worthwhile pausing to ponder the conceptual chasm between the intensional property and its extensional effects. A property like FTyp.ia/tr or Pos.L/R is about the ranking structure of grammars and enforces a relation between certain constraints. These constraints are not defined to dictate the shape of entire output forms: they merely accumulate penalties for certain structural configurations, often quite local, that occur within individual outputs. They do not (and cannot) say things like ‘every binary foot is iambic in all optima’, or ‘the unary and binary feet in optima are arranged in such-and-such a way’. On the extensional side, the various structural patterns appearing in optima show a distribution that may be quite distantly related to the types of items and relations monitored by the constraints. For this reason, we enforce a terminological distinction between intensional *properties* and extensional *traits*. The nature of these traits and their distribution emerges from the functioning of the grammar. For any OT system S, this involves the interplay of the constraints of $\text{C}\textsc{on}_{\text{S}}$, the candidate sets admitted by $\text{G}\textsc{en}_{\text{S}}$, and the definition of optimality that the theory is built from. Analysis rather than onomasty is required to establish the intensional-extensional connection.

When we understand the extensional traits associated with the intensional properties that generate a typology, we have a full description of the linguistic structure imposed by the grammars of the typology. This is essential to understanding what the typology says about linguistic form. A valid universal support delivers the entire set of *grammars* in a typology, but it is never more than a finite sample of what is typically an infinite collection of candidate sets. The distribution of extensional traits in a finite support may be suggestive of the broader pattern, but the mere finitude of the sample typically disallows secure generalization. The grammars must be examined, because they define the behavior of *every* cset. And the properties define the grammars.

From a property set for a typology, a *classification* or *property analysis* in the terminology of A&P, we can derive a complete description of the typology's extensional traits if we have a full analysis of the extensional consequences of each of the properties. A grammar is specified as a set of property values, and these induce a set of traits, which completely delimits the extensional structure of the optima for that grammar. Just as the grammar emerges from conjoining the property values, so does the description of its optimal forms emerge from conjoining the descriptions of traits associated with those values.

Aside from whatever desirability inheres in relating the grammars of a typology to their predictions about linguistic form, there is a further use for the full extensional interpretation of the intensional properties: it can verify that the typology was generated from a valid universal support. From a proposed support, we can mechanically calculate a set of grammars, which may be classified into a set of properties and their values. The validity and completeness of a proposed classification may easily be checked by simply running through the sets of value choices and verifying with the entailment algorithm of Prince 2002a:31ff that the ERC sets so derived are in a mutually entailing relation with the grammars produced by calculation; or the check may call on the algorithm FRed to show that the grammars produced by the property set are identical in MIB or SKB form with those produced by the support (see Brasoveanu & Prince 2011 for details).[4]

Verifying that the support is universal requires further analysis. A claimed support is authentically universal if the grammars it induces cannot be further refined. Refinement requires splitting a putative grammar by determining that it contains distinct legs that produce distinct optima from a single input. The P&S support is not universal because it asserts grammars for languages with (C)V(C) syllables that are split by their treatment of /C/.

The close association between properties and traits allows us to pursue the universality question in the extensional domain. When a typology is analyzed as a set of properties, each of its grammars is equivalent to a conjunction of (intensional) property values. These values entail

[4] FRed = "Fusional Reduction." MIB = "Most Informative Basis." SKB = "Skeletal Basis." The Fusional Reduction algorithm produces maximally concise ERC set representations of grammars ("bases"). The MIB includes all ranking information due to transitivity, the SKB is transitively reduced.

extensional traits. If every conjunction of property-value-derived *traits* yields the description of a *single* optimal candidate for each input admitted by $\text{G}\small{\text{EN}}_S$, then we can be sure that the grammars associated with the property values cannot be refined. (In the fully general case, which we will not encounter here, the extensional description can admit multiple candidate instantiations as long as they all have the same violation profile.) From a grasp of the intensional and extensional structure of the typology, we can prove the universality of a proposed support, or in the case of non-uniqueness, disprove it. In short, knowing how every grammar behaves extensionally — knowing every language — tells us whether we know of every grammar.

We carry out this program for the foot-based prosodic system nGX as analyzed by A&P (in prep.). The bulk of our argument develops a detailed analysis of the extensional traits associated with property values. We then show that each full, grammar-defining choice of values yields a description that delimits a single candidate for each distinct input. We conclude by using this value-based description to characterize the possible universal supports of nGX.

The method of proving universality developed here is general and may be applied to any well-defined OT system. The study of nGX illustrates how the basic logic of the theory, when honored, leads to certainty and understanding.

0.1. THE SYSTEM nGX AND ITS PROPERTIES. From A&P (in prep.), we have the following specification of the system nGX, which recognizes various aspects of stress patterning that do not depend on the presence of a main stress, on syllable weight, or on left-right asymmetries in metrical patterning.

(1) GEN_{nGX}

a. Inputs are strings of atomic units, representing syllables.
b. An input is associated with outputs of exactly the same length in syllables.
c. An output consists of a single Prosodic Word.
d. A Prosodic Word consists of feet and syllables.
e. A Foot consists of one or two syllables.
f. A syllable may belong to at most one foot.
g. A Foot has a unique head.
h. A Prosodic word has at least one foot.
i. The output set from an input contains every parse admitted by these requirements.

Certain traits of output forms of nGX are fixed by GEN_{nGX} and others are left open to vary between candidates. The units of nGX structure are easily and briefly enumerated: syllable, Foot, Prosodic Word. A syllable is parsed into a foot or not, in which case it is termed an *unparsed* syllable. A foot, monosyllabic or bisyllabic, contains a distinguished syllable, the head. The Prosodic Word is free to contain feet in any non-overlapping, non-recursive disposition, so long as at least one is present. To uniquely identify a specific output form, we need only indicate how long it is in syllables, where in the syllabic string each of its feet begins and ends, and which syllables have the status of head-of-foot. This description is unambiguous because there is literally nothing else to specify. This observation makes up in usefulness what it lacks in profundity, and is therefore worthy of recognition.

(2) **Remark**. **Unambiguous Description of nGX forms**. An output form of nGX is uniquely identified by its length, the location of its feet with respect to the syllable string, and the location of the heads of the feet.

To define CON$_{\text{nGX}}$, we introduce some notation. OTWorkplace (Prince, Tesar, & Merchant, 2007-2015) provides a convenient string-based spelling system for the parses of nGX and similar prosodic systems:

o	syllable not parsed into a foot (*unparsed* syllable)
u	nonhead of a foot
X	head of foot
-	edge of foot or unparsed syllable

This is easily seen to accord with the requirements of Remark (2), and therefore can be used to refer without fear of ambiguity to the hierarchical structures sanctioned by GEN$_{\text{nGX}}$. In addition, we will write 'f' for 'foot of any kind,' 'F' for 'binary foot,' 'U' for 'unary foot,' and 'σ' for 'syllable of any kind'. In constraint definitions, we use $\lessdot$ to mean 'precedes' and '$\gtrdot$' to mean 'follows'. With these notations, the constraints of nGX may be defined as follows, where * is the familiar OT star operator returning the number of matches in a candidate to the pattern specified after it. All constraints are markedness constraints; we therefore omit the prefix 'm'.

(3) CON$_{\text{nGX}}$

- Parse-σ (Ps) *o
- Iamb *-X
- Troch *X-
- AFL *{σ,f}: σ $\lessdot$ f
- AFR *{σ,f}: σ $\gtrdot$ f

The definitions of AFL and AFR are derived from those of Hyde (2012:803, 2015:22). We elide reference to the Prosodic Word as the domain of alignment, since it is the only domain admitted by GEN$_{\text{nGX}}$ that contains σ and f. On this approach, AFL accumulates a penalty for each pair {σ,f} in an output where σ precedes f. This provides an overall violation score for a form that effectively sums the distance in syllables of each foot f from the left edge of the Prosodic Word, which is exactly the intention of McCarthy & Prince (1993) for evaluation by this constraint, as first suggested by Robert Kirchner (p.c.). AFR does the same, in mirror image.

The name of the system acronymically encodes its key features. The definitions of Iamb and Troch are ***n***ew, in that they penalize feet by placement of the head: Iamb penalizes head-initial feet, Troch head-final feet. The effect is that both penalize unary feet. In some earlier conceptions, implicitly for example in P&S, the constraints penalized only -uX- as non-trochaic and -Xu- as non-iambic, treating unary feet as being *both* iambic and trochaic. Positioning of feet is done by ***G***eneralized Alignment (McCarthy & Prince 1993). All outputs consist of Prosodic Words with at least one foot: hence the suffixal ***X***.

With $\langle \text{G}{\small\text{EN}}_{nGX}, \text{C}{\small\text{ON}}_{nGX} \rangle$ defined, a set of 12 grammars, listed in Appendix 1, may be generated from a support that consists of two complete csets with inputs of length 3σ and 4σ. The same 12 may be generated from an input of length 5σ. This replication hints that 12 might be the actual number of grammars in the typology, and that they might be exactly the ones we have generated. Our goal is to show conclusively that this is true, and that both supports are universal.

Observe that we are not trying to show that the extensional languages of nGX are those of the world around us. We execute the prior task of determining what the grammars of nGX actually are, free of all heuristics, speculation, and unjustified belief. Our method securely connects the grammars of nGX with the unbounded extensional languages that they predict, providing the basis for further empirical and theoretical development.

A&P analyze the set of 12 grammars into 4 properties. We resist calling it the 'typology of nGX' until we have shown that it was generated from a universal support. The content of the properties is the subject of sections §1-4. A property value holds of every leg in a grammar that 'satisfies', 'meets', 'falls under' or 'is specified for' that value. The following table gives the *classification* of nGX that we will argue from. Its contents are described immediately below it.

(4) **Properties of nGX**

Name	Values	ERCish Form: Ps **.** Ia Tr **.** AFL AFR
FTyp.ia/tr	Iamb <> Troch	e.WL.ee / e.LW.ee
Pos.L/R	AFL <> AFR	e.ee.WL / e.ee.LW
Mult.D/sp	Ps <> ⟨F.sub, A.dom⟩	W.eL.LL / L.eW.WW W.Le.LL / L.We.WW
Un.X/o	Ps <> ⟨F.dom, A.dom⟩	W.LL.LL / L.WW.WW

Terminology and notation. The analysis develops within the theory of A&P, in which each property takes two contradictory values of the form $X \gg Y$ and $Y \gg X$, abbreviated X<>Y. Each property has a name N followed by the names of its values a and b, thus N.a/b. The names used here include the following mnemonically-chosen abbreviations:

(5) **Properties and Values**

Prop	Value	Abbreviates	
Ftyp	ia	foot type	iambic
	tr		trochaic
Pos	L	position (of feet)	left
	R		right
Mult	D	multiplicity (of feet)	dense
	sp		sparse
Un	o	unary constituents	unparsed syllable
	X		unary foot

The notation $\langle A,B\rangle <> \langle C,D\rangle$ abbreviates the following ranking requirements:

a. $A|B \gg C\&D$: 'either A or B dominates both C and D,' and

b. $C|D \gg A\&B$: 'either C or D dominates both A and B.'

The relation between the values follows the ERCish treatment of aggregates of constraints: disjunction of the dominators, conjunction of the subordinates, noted in fn. 1. The example may be represented as an ERC (fragment) WWLL and its opposite LLWW, where the constraints are listed in the arbitrary order A, B, C, D. The values of the second ERC are obtained by applying the rules of negation in ERC logic to the first. These are: ¬W=L, ¬L=W, ¬e=e. See Prince 2002a for detailed discussion and analysis.

In specifying the ranking relations between constraints, a property may refer not just to a specific constraint, like Iamb or AFL, but to a class of two (or more) constraints, picking out a certain member of that class in a given leg by virtue of its relation to other members of the class. For example, the constraints of the class F = {Iamb, Troch} are set against each other in the property FTyp: Iamb $<>$ Troch. But the properties Mult and Un call on the class F in a more indirect way, referring to a member of F as 'F.dom' or as 'F.sub', depending on whether it is the dominant or subordinate member of its class in a given linear order. The effect can be seen in these two examples:

(6) Dom and Sub constraints in legs

a. $\lambda_1 = $ *Iamb* $\gg$Ps $\gg$AFL$\gg$ *Troch*$\gg$AFR   F.dom(λ_1) = Iamb, F.sub(λ_1) = Troch

b. $\lambda_2 = $ *Troch*$\gg$Ps$\gg$ AFL$\gg$ *Iamb*$\gg$ AFR   F.dom(λ_2) = Troch, F.sub(λ_2) = Iamb

Similar remarks may be made of the class A = {AFL, AFR}.[5] In the present context, reference to dom/sub members of a constraint class allows us to look beyond the iambic/trochaic distinction

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[5] Expressions of the form X.dom and X.sub are therefore to be understood as functions on the set of linear orders, where X.dom(λ) returns the member of the set X that is ranked in λ above all other members of X occurring in λ. By the same token, X.sub(λ) returns the member of X that is ranked in λ *below* all other members of X. A property value written X.dom$\gg$ Y.dom is true of a grammar Γ iff for every $\lambda \in \Gamma$, X.dom(λ)$\gg$ Y.dom(λ), and similarly for expressions involving X.sub, Y.sub. Observe that on this conception, our example property $\langle A,B\rangle <> \langle C,D\rangle$ is {A,B}.dom$<>${C,D}.dom. See A&P (2015, in prep.) for details.

to recognize traits that generalize across the foot-types. For example, the legs just cited belong to different grammars, but are unified by the fact that they both select parses with as many feet as possible, up to binarity, regardless of whether the feet are iambic or trochaic. Further properties of the dom/sub operators emerge in other typologies examined in A&P (in prep.).

Property values classify grammars by shared ranking requirements and distinguish them by contradictory requirements. Each property stated above bifurcates the typology of nGX into those grammars that satisfy one value and those that satisfy the other. Thus, the 'scope' of each property — the set of grammars to which it is relevant — is the entire typology. In the case of Un.o/X, there is however a natural narrowing of scope which simplifies the property. If limited to Mult.D grammars, it becomes Ps<>F.dom, which sponsors opposing ERCs W.LL.ee / L.WW.ee. A&P refer to the phenomenon of limited scope as 'mootness', since the distinction made by the property is moot outside its scope. We show how this works in §4.1.

For convenience of reference, we assume a fixed but arbitrary sequencing of the $\text{C}\textsc{on}_{\text{nGX}}$ constraints in ERCS: Ps, Ia, Tr, AFL, AFR. We use dots to separate the constraints into the classes {Ps}, F = {Iamb, Troch}, A = {AFL, AFR}. This allows us to abbreviate ERCs as e.g. W.eL.Le, which represents Ps $\gg$ Tr & AFL: Ps dominates both Tr and AFL.

0.2. Mode of analysis. A grammar is specified by choice of property values. With properties from the inventory (5), we have grammars such as these, spelling out the values in the order Mult – FTyp – Pos – Un:

sp.ia.R.o
D.ia.L.o,
D.tr.R.X.

where each dotted slot in the sequence cites the name of a property value. Each value is associated with an ERC that holds of the grammar so defined. For example, Pos.L contributes the ERC e.ee.WL ('AFL$\gg$ AFR'), FTyp.ia contributes e.WL.ee ('Iamb$\gg$Troch'), and in accord with those choices, Mult.D contributes W.eL.Le , because Mult.D says 'Ps $\gg$ F.sub & A.dom', and in a grammar satisfying FTyp.ia and Pos.L, this spells out as 'Ps$\gg$ Troch & AFL'.

We investigate the extensional content of the properties by examining grammars that satisfy a certain property value or values. The same mode of argument is employed repeatedly. We analyze a competition between two candidates, call them **z** and **φ**, which are constructed so as to be in an informative relation. Competitors **z** and **φ** will typically differ in only one respect, thereby isolating the violation penalties contributed by the distinction between them. We determine the ERC **z**~**φ** by inspecting their relative performance on the constraints. Then we show that this ERC is entailed by the property or properties under consideration, employing the familiar inferential system of ERC logic (Prince 2002a, b). This establishes that **z** is *better than* **φ** in the grammar, which we write as '$\mathbf{z}\succ\boldsymbol{\varphi}$'. If $\mathbf{z}\succ\boldsymbol{\varphi}$ in a given grammar, then **φ** cannot be optimal under the conditions that the grammar meets, because there is always a better candidate, namely **z**. Recall that $\mathbf{z}\succ\boldsymbol{\varphi}$ is a strict order relation defined with respect to a specific grammar, and it

means that, for every leg λ of that grammar, λ selects **z** from the set {**z**,**φ**}. If a grammar satisfies the ERC **z**~**φ**, where **z** and **φ** differ in violation profile, we are guaranteed that **z**≻**φ**.
The force of the argument is that *no* candidates of the form of **φ** can be optimal, and by choosing our **φ**'s and **z**'s properly, we can rule out all structural types except the one that mirrors the extensional content of the property we are examining. This is the one hammer we need to smite all the suboptima.

This is a bounding argument, of a type generally familiar from the OT literature: see P&S (p.116-119, 209-212) for characterization and use. In the ordering of candidates imposed by the property value(s) under discussion, we show that **z** *bounds* **φ**, eliminating **φ**'s chances for optimality. This form of argument is commonly used to establish general *harmonic bounding*, which holds in every ranking. (P&S; Samek-Lodovici 1992, Samek-Lodovici and Prince 1999). We make use throughout of a more articulated form, focusing on the subset of rankings admitted by a property value or values, taking advantage of the resources of ERC logic. P&S (p. 262-4) prove aspects of their Onset Theorem and Coda Theorem (stated p.113-4) with similar arguments. Prince 2006 makes extensive use of ERC-logic entailment relations to establish properties of OT systems.

Certain of our conclusions require only harmonic bounding arguments, independent of the properties. These involve the distribution of unary feet in optima. From harmonic bounding alone we can show that an optimum must have at least one binary foot if it has enough syllabic material to support one: put another way, the feet of an optimal output of 2 syllables or greater in length cannot all be unary. To illustrate the character of a bounding argument, and to get a start on the characterization of extensional effects of the constraint system, we develop the argument here. We build up to the desired result through a sequence of two more specific lemmas, each established by a harmonic bounding argument. We use the U, F, and σ notation from above; foot edges, when invoked, are indicated by parentheses. We write |**α**| for the length in syllables of the form **α**. For any constraint C, we write C(**q**) for the numerical penalty assigned by C to candidate **q**, and thus an expression like C(**q**) < C(**z**) declares that one number is less than another.

(7) **Lemma**. No optimal output contains a sequence UU.

Proof. Consider any form **φ** = […UU…] and a competitor **z** = […F...] which is exactly like **φ** in every respect except that the syllables parsed UU in **φ** are parsed as F in **z**. To construct the ERC **z**~**φ**, we compare C(**z**) and C(**φ**) numerically over every constraint C∈$\text{C}{\small\text{ON}}_{nGX}$.

z = […F…], **φ** = […UU…]		
z~φ	Comparison of violation values	Rationale
e	Ps(**z**) = Ps(**φ**)	The status of syllables as footed or unfooted is the same in both.
W	Iamb(**z**) < Iamb(**φ**)	UU contributes 2 violations to Iamb(**φ**). If F is iambic, it contributes no violations to Iamb(**z**); if F is trochaic, it contributes 1 violation to Iamb(**z**). All other feet are the same in both competitors.
W	Troch(**z**) < Troch(**φ**)	The same reasoning applies mutatis mutandis.
W	AFL(**z**) < AFL(**φ**)	UU = (σ)(σ) and F = (σ σ). AFL sees UU as (σ (σ and F as (σ σ, where the left parentheses denote edges which, when non-initial, contribute violations to the AFL score. The UU structure in **φ** contains one more AFL-relevant foot edge than the F structure in **z** and that extra edge in **φ** is non-initial, guaranteeing at least one additional violation. All other feet are in the same positions in both.
W	AFR(**z**) < AFR(**φ**)	The same reasoning applies mutatis mutandis to UU = σ) σ) and F = σσ).

Given these numerical relations, the resulting ERC **z~φ** is e.WW.WW, indicating that **z** harmonically bounds **φ**. No form containing a sequence UU can be optimal. □

(8) **Lemma**. No optimal output contains a sequence Uo or oU.

Proof. Case 1**.** Let **φ** = […Uo…]. Consider a competitor **z** = […F...] which is exactly like **φ** in every respect except that the material parsed Uo in **φ** is parsed as F in **z**. To construct the ERC **z~φ**, we compare **z** and **φ** over every constraint.

$\mathbf{z}$ = […F…], $\boldsymbol{\varphi}$ = […Uo…]		
$\mathbf{z}$~$\boldsymbol{\varphi}$	Comparison of violation values	Rationale
W	Ps($\mathbf{z}$) < Ps($\boldsymbol{\varphi}$)	The form $\mathbf{z}$ has one fewer unparsed syllable than $\boldsymbol{\varphi}$.
e/W	Iamb($\mathbf{z}$) ≤ Iamb($\boldsymbol{\varphi}$)	If F is trochaic, we trade one non-iambic foot for another, and the scores are equal. If F is iambic, $\mathbf{z}$ has one less non-iambic foot than $\boldsymbol{\varphi}$.
W/e	Troch($\mathbf{z}$) ≤ Troch($\boldsymbol{\varphi}$)	The same reasoning applies mutatis mutandis.
e	AFL($\mathbf{z}$) = AFL($\boldsymbol{\varphi}$)	From the point of view of AFL, Uo = (σσ and F = (σσ, contributing equal numbers of violations to the AFL score.
W	AFR($\mathbf{z}$) < AFR($\boldsymbol{\varphi}$)	From the point of view of AFR, Uo = σ)σ and F = σσ). Since the cited foot edge in Uo in $\boldsymbol{\varphi}$ is one syllable farther from the end of the form than the cited edge in F, it contributes one more violation to the AFR score than F in $\mathbf{z}$.

The resulting ERC $\mathbf{z}$~$\boldsymbol{\varphi}$ takes the form W.We.eW or W.eW.eW, depending on whether F is iambic or trochaic. Thus $\mathbf{z}$ harmonically bounds $\boldsymbol{\varphi}$. No form containing Uo can be optimal.

Case 2. Let $\boldsymbol{\varphi}$ = […oU…] and $\mathbf{z}$ = […F…]. The reasoning here is the same as in Case 1, except that the roles of AFL and AFR are interchanged. The resulting ERCs take the form W.We.We or W.eW.We, depending on whether F is iambic or trochaic. Once again, $\mathbf{z}$ harmonically bounds $\boldsymbol{\varphi}$. No form containing a sequence oU can be optimal. □

(9) **Proposition**. **Binarity**. No optimal output longer than 1σ lacks binary feet.

Proof. Let $\boldsymbol{\varphi}\in\{\text{U,o}\}^+$ where $|\boldsymbol{\varphi}|$ is 2 syllables or greater. From $\text{G}{\scriptstyle\text{EN}}_{\text{nGX}}$, we have it that $\boldsymbol{\varphi}$ contains at least one foot. Consider some such form $\boldsymbol{\varphi}$ = […U…], where at least one of the stretches "…" is nonempty. U must be immediately followed or immediately preceded by a syllable to its left or by a syllable to its right. That neighboring syllable may be parsed as a unary foot, so the cited U is in a configuration UU. Or the neighboring syllable may be unparsed, so that cited U is in a configuration Uo or oU. There are no other cases. From Lemma (7) and Lemma (8), we know that no optima contain any of these configurations. Since $\text{G}{\scriptstyle\text{EN}}_{\text{nGX}}$ requires the presence of a foot in outputs, and since no $\boldsymbol{\varphi}\in\{\text{U,o}\}^+$ with $|\boldsymbol{\varphi}| \geq 2$ is optimal, the only optimal forms of length 2 syllables or longer have at least one binary foot. □

We conclude by noting a feature of ERC logic that we will take advantage of when convenient. In Boolean reasoning, if the antecedent is false, there is no need to worry about the truth value of the consequent: the same holds coordinatewise in ERC logic when an antecedent coordinate is L. Recall that entailment from one ERC to another depends on two rules of inference, L-retraction and W-extension (Prince 2002a:5-7). To obtain $\alpha\vDash\beta$, where α, β are logically nontrivial in that they each contain both W and L, it is necessary and sufficient that in each coordinate k we have $\alpha[k] \leq \beta[k]$, where the comparative values are arrayed along an abstract scale L<e<W. The rule of W-extension says that if the consequent coordinate value $\beta[k]$ = W, then $\alpha[k]$ may be anything. The rule of L-retraction says that if antecedent $\alpha[k]$ = L, then $\beta[k]$ may be anything. The upshot is

that in arguing $\alpha \vDash \beta$ coordinatewise, we can ignore those coordinates k where $\alpha[k]$ has L, because nothing in $\beta[k]$ can obstruct the entailment relation, by L-retraction.

(10) **Remark**. *Ex Falso Quodlibet*. For ERCs α and β, if $\alpha[k]$ = L, then in arguing $\alpha \vDash \beta$ by checking relations between values in the coordinates, we need not check the value of $\beta[k]$.

We will employ this handy shortcut on a couple of occasions below, to simplify the calculations when they threaten to distract.

0.3. CONSPECTUS OF THE ARGUMENT. Here we present the entire sequence of assertions that constitute the argument of the paper. The goal is to facilitate access to details and to overall structure by allowing the reader to view and review the exact steps that are taken along the path to the main conclusions. We eschew discursive paraphrase as obfuscatory and likely to provide a moral hazard.

To demarcate the relations between the various points in the landscape, we follow a usage that distinguishes between *lemmas*, which prove contributory results, *propositions*, which may depend on them, and *theorems*, larger results depending that may depend on both. With the exception of the analysis of unary elements, which depends on harmonic bounding and nothing else, the argument proceeds from property to property, showing in each case that the values of the property, set either way, exclude various substructures from appearing in optimal forms. The collective effect of the exclusions is to narrowly limit the shape that optimal forms may take. As the argument progresses, the extensional traits associated with property values bring the optima into tighter and tighter focus. By the end, we have achieved a complete extensional characterization of the optima, tied to the ranking requirements that underwrite their optimality, so that it becomes possible to show that each grammar delivers exactly one optimum from each cset admitted by the definition of nGX. This entails that any support leading to the 12 grammars generated by choice of property values is universal. In addition to this assurance, we have obtained detailed knowledge of the entire range of structural predictions of the theory: the extensional traits borne by optima and their derivation from the constraint interactions identified as crucial by the property analysis.

Unaries (§0.2)

- ● Lemma (7). No optimal output contains a sequence UU.
- ● Lemma (8). No optimal output contains a sequence Uo or oU.
- ■ Proposition (9). **Binarity**. No optimal output longer than 1σ lacks binary feet.

FTyp.ia/tr (§1)

- ■ Proposition (11). **Uniformity of Foot Type**. All the binary feet of an optimal output have the same type, iambic or trochaic.

Pos.L/R (§2)

● Lemma (12). Under Pos.L, no optimal output contains FU. Under Pos.R, no optimal output contains UF.

● Lemma (13). No optimal output contains a unary foot displaced from the dominant edge.

▸ Corollary (14). One U. No optimal output contains more than one U.

● Lemma (15). No optimal output under Pos.L contains the sequence oF. No optimal output under Pos.R contains the sequence Fo.

■ Proposition (16). **Positioning of Feet in Optima**. In a grammar satisfying Pos.L, any optimal output of length 2 or greater must be of the form $[(\mathrm{U})\mathrm{F}^n\mathrm{o}^k]$, $n \geq 1$, $k \geq 0$. In any grammar satisfying Pos.R, any optimal output of length 2 or greater must be of the form $[\mathrm{o}^k\mathrm{F}^n(\mathrm{U})]$.

Mult.D/sp (§3)

● Remark (19). Shrift for the Short. Optimal outputs of length 1 are of the form [U]. Optimal outputs of the length 2 are of the form [F].

Mult.sp

● Lemma (20). Under Mult.sp = ⟨A.dom, F.dom⟩≫Ps, no optimal output contains FF.

● Lemma (21). Under Mult.sp, no optimal output contains UF or FU.

■ Proposition (22). **The shape of the sparse**. In grammars satisfying Mult.sp = ⟨A.dom, F.sub⟩≫ Ps, all optima of length 2 or greater take the form $[\mathrm{Fo}^k]$ under Pos.L and $[\mathrm{o}^k\mathrm{F}]$ under Pos.R, $k \geq 0$.

Mult.D

● Lemma (23). Under Mult.D, no optimal output contains oo.

● Lemma (24). Under Mult.D, no optimal output contains both U and o.

■ Proposition (25). **The shape of the dense**. In grammars satisfying Mult.D = Ps≫ ⟨A.dom,F.sub⟩, all optimal outputs of length 2 or greater take the form $[\mathrm{F}^n(\mathrm{o})]$ or $[(\mathrm{U})\mathrm{F}^n]$ under Pos.L and $[(\mathrm{o})\mathrm{F}^n]$ or $[\mathrm{F}^n(\mathrm{U})]$ under Pos.R, $n \geq 1$.

Un.o/X (§4)

Un.X

● Lemma (27). Un.o/X and Sparseness. Sparse grammars cannot assume the value Un.X. Sparse grammars must assume the value Un.o.

● Lemma (28). Under Un.X, no optimal output contains o.

■ Proposition (29). **The shape of Un.X.** Under Un.X = Ps ≫ ⟨F.dom, A.dom⟩, all optimal outputs of length 2 or greater take the form $[(\mathrm{U})\mathrm{F}^n]$ under Pos.L and $[\mathrm{F}^n(\mathrm{U})]$ under Pos.R, $n \geq 1$.

Un.o

● Lemma (30). Under Un.o, no optimal output of length 2 or greater contains U.

■ Proposition (31). **The shape of Un.o**. In grammars satisfying Un.o, optima are of the form $[\mathrm{F}^n\mathrm{o}^k]$ under Pos.L and $[\mathrm{o}^k\mathrm{F}^n]$ under Pos.R, $n \geq 1$, $k \geq 0$.

Extensional Traits of the Optima of nGX (§5)

■ **Theorem** (33). **Optimal Outputs of nGX**. The optimal outputs of nGX of length greater than or equal to 2 syllables are drawn from the following patterns. Within each schema, F is uniformly iambic or uniformly trochaic, and $n \geq 1$, $k \geq 0$.

Mult/Un values	Pos.L	Pos.R
Mult.sp, (Un.o)	Fo^k	$\mathrm{o}^k\mathrm{F}$
Mult.D, Un.o	F^n(o)	(o)F^n
Mult.D, Un.X	(U)F^n	F^nU

Universal Supports for nGX (§6)

■ **Theorem** (34). **Universal Supports for nGX**. Any collection of candidate sets that delivers the 12 grammars obtained from the properties (4) is a universal support for nGX.

● Lemma (35). The Long Supports. Any odd-length input of 5 or more syllables provides a universal support for nGX.

● Lemma (36). Failure of the Even. No even-length input provides a universal support.

● Lemma (37). The support with two inputs, one of length 3, the other of length 2*m*, $m \geq 2$, is universal and minimal**.**

■ **Theorem** (38). **Minimal Universal Supports for nGX**. The minimal universal supports for nGX are (1) the csets from an input of length 3 and an input of length 2*m*, $m \geq 2$**,** and (2) any single cset from an input of odd length 5 syllables or greater. There are no others.

1. FOOT TYPE: FTyp.ia/tr. IAMB <> TROCH

We begin by establishing the extensional effects of the FTyp.ia/tr property.

(11) **Proposition**. **Uniformity of Foot Type**. All the binary feet of an optimal output have the same type, iambic or trochaic. The binary Foot type of the entire output form is determined by FTyp (F.dom $\gg$ F.sub).

Proof. Assume FTyp.ia = e.WL.ee, so that F.dom = Iamb. Consider any output **φ** that contains a binary foot $\bar{\mathrm{F}}$ of the subordinate trochaic type, so that **φ** = [… $\bar{\mathrm{F}}$ …]. Now consider an output **z** = […F…] which is exactly the same as **φ** except that the cited trochaic foot $\bar{\mathrm{F}}$ is replaced by iambic F. Claim: **z**≻**φ** in any grammar meeting FTyp.ia. To show this, we construct the ERC **z**~**φ**. Observe that both **z** and **φ** fare exactly the same on the constraints Ps, AFL, AFR. This leaves only Iamb and Troch to distinguish foot type, as claimed.

The form **φ** has one more non-iambic foot than **z**, so that Iamb(**z**) < Iamb(**φ**). On Troch, we have Troch(**z**) > Troch(**φ**), because **z** has one more non-trochaic foot than **φ**. We therefore have the following ERC:

	Ps	Iamb	Trochee	AFL	AFR
z~**φ**	*e*	W	L	*e*	*e*

We abbreviate this as **z**~**φ** = e.WL.ee.

Any grammar satisfying FTyp.ia = e.WL.ee meets this condition. The ERC **z**~**φ** is trivially entailed by FTyp.ia (since α⊨α), and we therefore have **z**≻**φ**. This means that **φ** cannot be optimal in any grammar satisfying FTyp.ia, because there is always a competitor better than **φ**. Any form like **φ** that contains even a single trochaic foot $\bar{\text{F}}$ has a competitor that betters it by virtue of lacking that foot. Whether the competitor is itself bettered by something else is immaterial to deducing the fate of **φ**. Therefore, in grammars under FTyp.ia, no trochaic binary foot appears in an optimum.

The argument holds, mutatis mutandis, for FTyp.tr when we simply interchange the roles of Iamb and Troch.

Thus no form with any subordinate-type binary feet is optimal, and each optimum must be chosen from among those with only dominant-type binary feet, as claimed. □

2. FOOT POSITION. Pos.L/R: AFL <> AFR

We now establish a number of facts about the positioning of feet in nGX.

(12) **Lemma**. Under Pos.L, no optimal output contains FU. Under Pos.R, no optimal output contains UF.

Proof. Assume that the grammar under consideration satisfies Pos.L = AFL≫AFR. Let **φ** = […FU…] and consider a competitor **z** = […UF…], identical to **φ** in every respect except for the parsing of the three syllables dominated by the cited F and U nodes. We calculate the ERC **z**~**φ**. Observe first that **φ** and **z** fare identically on the constraints Ps, Iamb, and Trochee. This gives us **z**~**φ** = e.ee.XY, where X and Y are values to be determined. Any distinction between **φ** and **z** is made entirely by AFL and AFR.

Notating foot-edges with parentheses, we observe that the original sequence FU has the structure (σσ)(σ). Only the left edges count for AFL, so the contribution of UF and FU to the overall AFL penalties is determined by the following reduced structures:

FU = (σ σ (σ *in* **φ** = […FU…]
UF = (σ (σ σ *in* **z** = […UF…].

These share whatever AFL penalty is induced by the leftmost foot-edge, but in **z** the second foot-edge is closer to the beginning of the word, thereby shaving one violation from its AFL score. Since everything else in forms **φ** and **z** is identical, we have AFL(**z**) < AFL(**φ**).

Applying the same reasoning to AFR, we have the following relevant representations:

FU = σ σ) σ) *in* **φ** = […FU…]
UF = σ) σ σ) *in* **z** = […UF…].

These share the AFR penalty induced by the second foot-edge, but in **φ** the first foot-edge is one syllable closer to the right edge of the word. Thus, AFR(**φ**) < AFR(**z**).
Putting these results together, we obtain the ERC e.ee.WL. Since Pos.L gives us exactly the same ERC, we have trivially that Pos.L⊨**z**~**φ**. Therefore, in any grammar satisfying Pos.L, we have **z**≻**φ**, and **φ** cannot be optimal.

The argument with respect to Pos.R and UF proceeds identically, exchanging right and left. This establishes the lemma. □

(13) **Lemma**. No optimal output contains a unary foot displaced from the dominant edge.

Proof. Consider grammars satisfying Pos.L, and consider a form **φ** = […U…]. If the first '…' is nonempty, displacing the cited unary from the dominant (left) edge, then the cited U must sit in one of three configurations: oU, UU, FU. The first two of these are not present in optima by Lemmas (8) and (7) respectively. The third is absent from optima under Pos.L according to Lemma (12). If the cited U is displaced from the left edge, then **φ** is not a possible optimum. The same argument may be replicated for Pos.R with mirror-image forms. □

(14) **Corollary**. **One U**. No optimal output contains more than one U.

Proof. Even if one U sits at the dominant edge, any other U must be displaced from the dominant edge, a guarantee of suboptimal status, by Lemma (13). □

(15) **Lemma**. No optimal output under Pos.L contains the sequence oF. No optimal output under Pos.R contains the sequence Fo.

Proof. Consider any grammar satisfying Pos.L. Let **φ** = […oF…]. Consider a competitor **z** in which the syllables parsed by the cited sequence oF in **φ** is parsed instead as Fo in **z**, so that we have **z** = […Fo…], where the competitors **φ** and **z** are identical in every respect outside the cited oF and Fo sequences. Since both **z** and **φ** consist of exactly the same units, they fare the same on Ps, Iamb, and Trochee and differ only on the alignment constraints.

We have the following AFL-relevant representations:

oF = σ (σ σ   *in* **φ** = […oF…]

Fo = (σ σ σ   *in* **z** = […Fo…].

The cited (left) foot-edge contributes one more violation to the score for AFL in **φ** than in **z**. Since everything else outside the cited sequences is the same in **z** and **φ**, we have AFL(**z**) < AFL(**φ**).

For evaluation by AFR, we have the following relevant representations:

oF = σ σ σ)   *in* **φ** = […oF…]

Fo = σ σ) σ   *in* **z** = […Fo…].

The cited (right) foot-edge contributes one violation less to the score for AFR in **φ** than in **z**. Since the two outputs are identical except for the cited sequences, it follows that AFR(**φ**) < AFR(**z**).

These lucubrations give us **z**~**φ** = e.ee.WL. Since this is the same as Pos.L, we have Pos.L⊨e.ee.WL, trivially. In any grammar satisfying Pos.L, we have **z**≻**φ**. It follows that **φ** cannot be optimal under Pos. L. This means that optimal outputs in Pos.L grammars cannot contain the sequence oF.

The same reasoning applies to grammars satisfying Pos.R, using mirror image forms. The result is that optima under Pos.R cannot contain Fo. □

(16) **Proposition**. **Positioning of Feet in Optima**. In any grammar satisfying Pos.L, any optimal output of length 2 or greater must be of the form $[(U)F^n o^k]$, $n \geq 1$, $k \geq 0$. In any grammar satisfying Pos.R, any optimal output of length 2 or greater must be of the form $[o^k F^n (U)]$.

Proof. Forms of length 2 are shaped [F] by Lemma (9), which guarantees the presence of at least one binary foot, meeting the claimed descriptions. Consider forms of length 3 or greater in grammars under Pos.L. If there is a unary foot in an optimal output, it cannot be displaced from the beginning, by Lemma (13). This means that only forms [(U)…], where '…' contains no

U, have any hope of optimality. If the form has a binary foot, and it must have at least one by Lemma (9), then no such foot can appear in optima in the configuration oF, by Lemma (15). This leaves only the configuration [(U)F^n… for binary feet to occupy in optima. Completing the form with unparsed syllables runs afoul of no condition established so far, so [(U)F^{n}o^k] is the only pattern left for optimal outputs to assume.

The same argument may be replicated for Pos.R mutatis mutandis. □

The optimal shapes established in Proposition (16) will be further refined as we proceed.

We conclude with a couple of remarks of more general interest. First, observe that *only* AFL and AFR are involved in positioning decisions in nGX. If two competing outputs have exactly the same number of feet of exactly the same type, they perform identically on the constraints Ps, Iamb, Troch, as noted in the arguments above. They can only be distinguished by foot location, which is monitored by AFL and AFR.

Second, and perhaps less obviously, in any contest between an optimum and a competitor that matches it in every trait except foot position, it is the dominant alignment constraint that decides between them. There are no cases where two such competitors, one optimal, both fare equally on the dominant constraint, leaving the decision to its subordinate antagonist. This is implicit in the proofs already given, and need merely be brought out.

(17) **Remark**. The dominant alignment constraint decides foot-location in competitions between an optimal form and a competitor with the same number and types of feet. The subordinate constraint makes no decisions.

Proof. By Proposition (16), all possible optima have either the shape [(U)F^{n}o^k] or the shape [o^{k}F^n(U)], $n\geq 1$, $k\geq 0$. We consider only forms of length 3 or greater. In forms of length 1, there is only one output admitted by GEN$_{\text{nGX}}$, so no competition. The same is true of length 2, because the only optima are shaped [F].

Consider the situation under Pos.L, and let **z** be an output possibly optimal under Pos.L, and therefore of the form [(U)F^{n}o^k], $n\geq 1$, $k\geq 0$. Observe that **z** must contain at least one binary foot, by Proposition (9), motivating the requirement that $n\geq 1$. Let $\boldsymbol{\varphi}$ be a competing parse, $\boldsymbol{\varphi}\neq\mathbf{z}$, with exactly the same number and types of feet as **z**. We make no supposition as to whether $\boldsymbol{\varphi}$ is possibly optimal, although by assumption there is an arrangement of its feet that is, namely **z**.

The prosodic pattern of **z** is the only arrangement of its prosodic units that lacks configurations oU, Uo, oF, or FU. Therefore $\boldsymbol{\varphi}$ must contain at least one of these. Since $\boldsymbol{\varphi} \neq$ [Uo^k], if $\boldsymbol{\varphi}$ contains Uo, it must also contain either oU, oF, or FU; we may therefore set [Uo^k] aside.

The proofs of Lemmas (12) and (15) establish that for any form $\boldsymbol{\varphi}$ containing a configuration FU or oF, respectively, we can produce another form *strictly* better than $\boldsymbol{\varphi}$ on AFL by virtue of lacking one of those configurations, namely by replacing them with UF and Fo respectively. The same reasoning applied to oF in Lemma (15) extends to oU. Thus, for any form $\boldsymbol{\varphi}$ which differs from **z** only in disposition of feet we can construct a $\boldsymbol{\varphi}'$ that lacks one of the instances oU, oF, FU, with AFL($\boldsymbol{\varphi}'$)<AFL($\boldsymbol{\varphi}$). But if $\boldsymbol{\varphi}'$ is itself not identical to **z,** then it must also contain at least one instance of oU, Uo, oF, or FU. Once again, any form containing Uo must also contain at least one of the others. We may therefore iterate the same reasoning until we

reach a form to which it does not apply, by virtue of lacking oU, Uo, oF, FU. But this can only be **z**. Therefore AFL(**z**) < AFL(**φ**). This shows that the decision between the possible optimum **z** and any competitor with the same prosodic units differently disposed is made entirely by AFL in grammars under Pos.L.

The same reasoning applies mutatis mutandis to mirror-image forms under Pos.R. □

3. FOOT MULTIPLICITY: Mult.D/sp. Ps<>⟨A.dom, F.sub⟩

The number of feet in optima is determined by Mult.D/sp. We introduce two useful descriptive terms, defined extensionally.

(18) **Sparse and dense**

a. A *sparse* language has exactly 1 foot in every output.
b. A *dense* language has more than 1 foot in *some* output forms.

These definitions suffice to distinguish the relevant classes of extensional languages in nGX. The terms are echoed in the values sp/D associated with the property Mult, but the value names refer to specific ranking relations, not to the traits of forms. The extensional content of the values Mult.sp and Mult.D will prove to be much more detailed than the broad distinction incorporated in the definitions of 'sparse' and 'dense' just given.

We begin the argument by noting that short optima have limited structural options.

(19) **Remark**. **Shrift for the Short**. Optimal outputs of length 1 are of the form [U]. Optimal outputs of the length 2 are of the form [F].

Proof. By GEN_{nGX} all outputs must contain at least one foot, settling the first case. In the case of length 2, observe that Lemma (7) rules out UU, and Lemma (8) rules [Uo] and [oU]. This leaves only [F]. □

3.1. THE SHAPE OF THE SPARSE: Mult.sp. ⟨A.dom, F.sub⟩≫ Ps. We first consider the extensional effects of Mult.sp.

(20) **Lemma**. Under Mult.sp, no optimal output contains FF.

Proof. Let **z** = […Foo…] and **φ** = […FF…], where **z** and **φ** are identical in every respect outside the specified regions FF and Foo. We may also assume, without losing sight of any possible optima, that all binary F are of the dominant foot type, as Proposition (11) assures us, which we will assume to be iambic by Ftyp.ia. We assume A.dom = AFL by Pos.L. We have, then, Mult.sp = ⟨Troch, AFL⟩≫Ps, ERCwise L.eW.We. As always, we want to examine the ERC **z**~**φ**, so we determine the relative performance of these competitors over the constraint set.

z = […Foo…], **φ** = […FF…]		
z~φ	Comparison of violation values	Rationale
L	Ps(**z**) > Ps(**φ**)	Form **z** has 2 more unparsed syllables than **φ**.
e	Iamb(**z**) = Iamb(**φ**)	All feet are identical in both, except for the cited sequences, which contain no non-iambic feet.
W	Troch(**z**) < Troch(**φ**)	Form **z** contains one fewer non-trochaic foot than **φ**.
W	AFL(**z**) < AFL(**φ**)	AFL sees Foo in **z** as (σ σ σ σ, and FF in **φ** as (σ σ (σ σ. In **φ**, the rightmost cited foot-edge incurs a penalty for AFL unmatched in **z**. The leftmost cited foot-edge in **φ** and in **z** incur the same AFL penalty.
e/W	AFR(**z**) ≤ AFR(**φ**)	AFR sees Foo in **z** as σ σ) σ σ and FF in **φ** as σ σ) σ σ). The leftmost cited foot-edge is identically placed in both. The rightmost foot-edge in **φ** incurs a positive penalty if FF is not final; if it is final in **φ**, it incurs no penalty.

Putting these considerations together, we have the ERC L.eW.We or L.eW.WW. Under FTyp.ia and Pos.L, Mult.sp = L.eW.We, which is identical to the first and asymmetrically entails the second by W-extension. Therefore, {FTyp.ia, Pos.L, Mult.sp}⊨**z~φ**, and we have **z**≻**φ** in grammars meeting these conditions. It follows that no form that like **φ** contains FF can be optimal in these grammars.

The argument replicates mutatis mutandis for the other values of FTyp.ia/tr and Pos.L/R, establishing the claim. □

(21) **Lemma**. Under Mult.sp, no optimal output contains UF or FU.

Proof. Assume for concreteness FTyp.ia and Pos.L. Lemma (13) guarantees that no optimum under Pos.L contains a unary foot displaced from the left edge, ensuring a fortiori that FU cannot appear in optima under the combination of Mult.sp and Pos.L. We therefore need only consider a form **φ** = [UF…] and contrast it with a form **z** = [Fo…], where the material in '…' is identical in the two forms.

z = [Fo…], **φ** = [UF…]		
z~φ	Comparison of violation values	Rationale
L	Ps(**z**) > Ps(**φ**)	Form **z** has one more unparsed syllable than **φ**.
W	Iamb(**z**) < Iamb(**φ**)	Form **φ** has one more non-iambic foot than **z**, namely the cited U, contributing 1 more violation of Iamb than **z** has.
W	Troch(**z**) < Troch(**φ**)	Form **φ** has one more non-trochaic foot than **z**.
W	AFL(**z**) < AFL(**φ**)	AFL sees Fo in **z** as (σ σ σ and the sequentially corresponding syllables in UF in **φ** as (σ (σ σ. The rightmost foot-edge in **φ** incurs a penalty on AFL not matched in **z**.
W	AFR(**z**) < AFR(**φ**)	AFR sees the UF structure in **φ** as σ) σ σ) and the Fo structure in **z** as σ σ) σ. The leftmost cited foot-edge in **φ** incurs a greater penalty on AFR than the only cited foot-edge in **z**.

Putting these observations together yields the ERC L.WW.WW which is clearly entailed by Mult.sp = L.eW.We. We have therefore {FTyp.ia, Pos.L, Mult.sp} ⊨**z**~**φ**. Thus **z**≻**φ**, and it follows that **φ** cannot be optimal in any grammar meeting these conditions.

The same reasoning applies mutatis mutandis when other values are chosen for FTyp.ia/tr and Pos.L/R, re-defining A.dom and F.dom. In particular, analysis of mirror image forms under Pos.R shows that sparse optima cannot contain FU. □

(22) **Proposition**. **The shape of the sparse**. In grammars satisfying Mult.sp = ⟨A.dom, F.sub⟩≫Ps, all optima of length 2 or greater take the form $[\mathrm{Fo}^k]$ under Pos.L and $[\mathrm{o}^k\mathrm{F}]$ under Pos.R, $k \geq 0$.

Proof. Optima are restricted to the forms $[(\mathrm{U})\mathrm{F}^n\mathrm{o}^k]$ and $[\mathrm{o}^k\mathrm{F}^n(\mathrm{U})]$, $n \geq 1$, $k \geq 0$, under Proposition (16). By Lemma (20) no optimal output under Mult.sp contains FF, and by Lemma (21) no optimal output under Mult.sp contains UF or FU. Sparse optima can then only be of the form $[\mathrm{Fo}^k]$ and $[\mathrm{o}^k\mathrm{F}]$. □

3.2. THE SHAPE OF THE DENSE: Mult.D. PS ≫ ⟨A.DOM, F.SUB⟩. From the property statement itself, we have Mult.D = Ps≫ ⟨A.dom, F.sub⟩. Setting as always F.dom = Ia and A.dom = AFL for ease of argument, we find ourselves dealing with grammars that satisfy the ERC W.eL.Le, which says Ps≫⟨AFL,Troch⟩. Fusing this with Pos.L = e.ee.WL, we have Pos.L∘Mult.D = W.eL.LL.[6] We now proceed to deduce extensional traits from this ERC, making use of the logical fact recorded as Remark (10) that if a coordinate value is L, we have nothing to verify about the corresponding coordinate in any ERC that we want it to entail. We can content

[6] On the logic of ERC fusion, see Prince 2002a,b *et seq*. Fusion of two ERCs combines their values componentwise, according to the scheme L∘X=X∘L=L; *e*∘X=X∘*e*=X; W∘W=W, for X∈{W,L,*e*}. We use the following fact: if the fusion of a set of ERCs entails an ERC α, then their logical conjunction entails α as well (Prince 2002a:10).

ourselves with establishing ERC fragments of a form that we can represent as W.e*x*.*xx* and W.W*x*.*xx* where the *x*'s indicate unknown values, which of course needn't be the same.

(23) **Lemma.** Under Mult.D, no optimal output contains oo.

Proof. Let **φ** = […oo…]. We compare with the minimally differing **z** = […F…], where F is of the dominant foot type, and where the sequentially corresponding '…' are structurally identical in **z** and **φ**. We assume for concreteness FTyp.ia, so that F is iambic in optima.

z = […F…], **φ** = […oo…]		
z~φ	Comparison of violation values	Rationale
W	Ps(**z**) < Ps(**φ**)	Form **z** has 2 fewer unparsed syllables than **φ**.
e	Iamb(**z**) = Iamb(**φ**)	Neither F nor oo contributes violations of F.dom = Iamb.

Of the ERC **z~φ**, we now know the crucial values. From **z~φ** = W.*ex*.*xx* and Pos.L∘Mult.D = W.eL.LL, we may conclude, in light of Remark (10), that

Pos.L∘Mult.D⊨**z~φ**

and therefore that **z≻φ**. The argument replicates mutatis mutandis for all other settings of the Pos.L/R and FTyp.ia/tr. Therefore, no output […oo…] may be optimal under Mult.D. □

The final restriction we need to characterize dense footing bans the co-occurrence of U and o in a single word, providing the one case where nonlocality enters into the argument.

(24) **Lemma**. Under Mult.D, no optimal output contains both U and o.

Proof. With F.dom = Iamb and A.dom = AFL, any optimal output must be of the form [(U)$F^n o^k$], with all F iambic, by Proposition (11) and Proposition (16). From Lemma (23), we know that $k \leq 1$, since dense outputs cannot contain oo. Therefore, any optimal output under Mult.D must be of the form [(U)F^n(o)]. Consider a U-containing form **φ** that contains both U and o. We must have **φ** = [UF^no]. This form cannot be of odd length, so we conclude that no odd length dense form may contain both U and o.

Now consider the competitor **z** = [F^{n+1}]. We have these relations on Ps and Iamb.

z = [F^{n+1}], **φ** = [UF^no]		
z~φ	Comparison of violation values	Rationale
W	Ps(**z**) < Ps(**φ**)	Form **φ** contains an unparsed syllable while **z** does not.
W	Iamb(**z**) < Iamb(**φ**)	Form **φ** contains the non-iambic foot U while **z** does not.

This gives us as much of the ERC **z~φ** as we need. We have **z~φ** = W.W*x*.*xx*. Since Pos.L∘Mult.D = W.eL.LL, we have

Pos.L∘Mult.D⊨**z~φ**

This establishes that **z≻φ**, and that therefore **φ** cannot be optimal. No form under Pos.L, FTyp.ia, and Mult.D may contain both U and o.

As always, these arguments may be reproduced under all choices of values for FTyp.ia/tr and Pos.L/R. It follows that no optimal outputs of any length, odd or even, may contain both U and o under Mult.D. ☐

(25) **Proposition**. **The shape of the dense**. In grammars satisfying Mult.D = Ps ≫ ⟨A.dom,F.sub⟩, all optimal outputs of length 2 or greater take the form [F^n(o)] or [(U)F^n] under Pos.L and [(o)F^n] or [F^n(U)] under Pos.R, n≥1.

Proof. Optima are restricted by Proposition (16) to the forms [(U)$F^n o^k$] and [$o^k F^n$(U)]. Under Mult.D, optimal outputs may not contain oo by Lemma (23), nor may they contain both U and o by Lemma (24), so under Pos.L the only dense optima have the shapes [F^n(o)], [UF^n]. Under Pos.R, the possible optima must be shaped as their mirror images. ☐

(26) **Corollary**. All dense even-length outputs are exhaustively parsed into binary feet.

Proof. Forms [F^no] and [UF^n] are of odd length. This leaves only [F^n] among the patterns of optima admitted by Proposition (25). ☐

4. SYLLABLES OUTSIDE BINARY FEET: Un.o/X. ⟨A.dom, F.dom⟩<> Ps.

All that remains to be determined is whether a dense odd-length output has an unparsed syllable or a unary foot. The property is determined by Un.o/X: ⟨A.dom, F.dom⟩ <> Ps.

(27) **Lemma**. **Un.o/X and Sparseness**. Sparse grammars cannot assume the value Un.X. Sparse grammars must assume the value Un.o.

Proof. We show that the values Un.X and Mult.sp are contradictory.

[1] Mult.sp = ⟨A.dom, F.sub⟩ ≫ Ps.

[2] F.dom ≫ F.sub.

[3] Mult.sp ⊨⟨A.dom, F.dom⟩≫Ps = Un.o, from [1],[2] by transitivity of domination.

[4] Un.X = Ps ≫ ⟨A.dom, F.dom⟩.

[5] {Mult.sp,Un.X}⊨⟨A.dom,F.dom⟩≫ Ps, and Ps≫ ⟨A.dom,F.dom⟩, from [3], [4].

Recall that '⟨A,B⟩≫C' means ' (A≫C) *or* (B≫C) ', while 'C≫⟨A,B⟩' is '(C≫A) *and* (C≫B)'. Domination is a strict order, and therefore asymmetric, so [5] is a contradiction. This leaves only the value Un.o = ⟨A.dom, F.dom⟩≫ Ps to be available for conjunction with Mult.sp. Un.o is not only consistent with Mult.sp = ⟨A.dom, F.sub⟩≫Ps, but is logically entailed by it, by line [3]. ☐

The choice between the values of Un.o/X is therefore only relevant to the grammars of the dense languages.

(28) **Lemma**. Under Un.X, no optimal output contains o.

Proof. Un.X = Ps≫⟨A.dom, F.dom⟩. Since A.dom≫A.sub and F.dom≫F.sub, Un.X delivers the ERC W.LL.LL no matter which values of Pos.L/R and FTyp.ia/tr are chosen. Let **z** be any form that is fully parsed, and **φ** a competitor with at least one unparsed syllable. Clearly **z**~**φ** = W.*xx.xx*. By the logic of Remark (10), this is entailed by W.LL.LL no matter what values are assigned to the individual x's, so under Un.X we have $\mathbf{z} \succ \boldsymbol{\varphi}$, eliminating $\boldsymbol{\varphi}$'s chances for optimality, and with it the chances of any form with unparsed syllables. □

(29) **Proposition. The shape of Un.X.** In grammars satisfying Un.X = Ps $\gg$ ⟨F.dom, A.dom⟩, all optimal outputs of length 2 or greater take the form [(U)F^n] under Pos.L and [F^n(U)] under Pos.R, $n \geq 1$.

Proof. From Lemma (27), no Mult.sp grammar can assume the value Un.X, so all Un.X grammars can only be Mult.D. Indeed, Un.X = Ps $\gg$ ⟨A.dom, F.dom⟩ entails Mult.D = Ps$\gg$⟨A.dom, F.sub⟩ because F.dom $\gg$ F.sub. From Proposition (25), under Mult.D, optima are of the form [F^n(o)] or [(U)F^n] assuming Pos.L, and the mirror images under Pos.R. From Lemma (28), in all grammars satisfying Un.X, optimal outputs are completely parsed, eliminating outputs of the form [F^no] and [oF^n].

This leaves only [(U)F^n] under Pos.L and [F^n(U)] under Pos.R. □

(30) **Lemma**. Under Un.o, no optimal output of length 2 or greater contains U.

Proof. Un.o = ⟨A.dom, F.dom⟩$\gg$Ps. From Lemma (27), line [3], Mult.sp ⊨ Un.o. Grammars under Mult.sp allow no U in optimal outputs of length greater than 1 syllable by Proposition (22), so this class of grammars under Un.o satisfies the statement of the lemma. We turn now to the remaining class of grammars, those satisfying Mult.D. Consider any dense form $\boldsymbol{\varphi}$ = [UF^n] and a dense competitor $\mathbf{z}$ = [F^no]. Setting A.dom = AFL, F.dom = Iamb, we have the following ERC from fusion: Mult.D◦Un.o◦FTyp.ia◦Pos.L = L.WL.LL.

Explicitly,

Mult.D	`W.eL.Le`
Un.o	`L.We.We`
FTyp.ia	`e.WL.ee`
Pos.L	`e.ee.WL`
fuse(all)	`L.WL.LL`

By Remark (10), we need only show that F.dom (Iamb) assigns fewer violations to $\mathbf{z}$, thus ensuring a W in the second coordinate of the ERC $\mathbf{z}$~$\boldsymbol{\varphi}$. Forms $\boldsymbol{\varphi}$ = [UF^n] and $\mathbf{z}$ = [F^no] have the same number of binary feet, which we may assume to be of the dominant, iambic type. But $\boldsymbol{\varphi}$ contains an additional unary foot, adding one to its Iamb penalty. This gives the ERC x.Wx.xx, which is entailed by Mult.D◦Un.o◦FTyp◦Pos.L = L.WL.LL. In any such grammar, $\mathbf{z} \succ \boldsymbol{\varphi}$. Parallel arguments hold by switching the values of F.dom and A.dom. Switching A.dom does not affect the fused ERC. Switching F.dom gives us the ERC L.LW.LL, and the same form of argument goes through under the assumption that F in the competitors is trochaic. □

(31) **Proposition. The shape of Un.o.** In grammars satisfying Un.o, optima are of the form [$F^n o^k$] under Pos.L and [$o^k F^n$] under Pos.R, $n \geq 1$, $k \geq 0$.

Proof. Consider first grammars satisfying Mult.sp. By Lemma (27), line [3], these must have the value Un.o. From Proposition (22), we have that in all such grammars optima are of the form [Fo^k] and [o^kF], which accords with the claim that Un.o grammars admit only [$F^n o^k$] and [$o^k F^n$] under Pos.L and Pos.R respectively.

Now consider grammars satisfying Mult.D. By Proposition (25), all optima in Mult.D grammars are of the form [(U)F^n] and [F^n(o)] under Pos.L and the mirror images under Pos.R. But by Lemma (30), no optimal outputs of length 2 or greater contain U under Un.o. Therefore only [F^n(o)] and [(o)F^n] remain. These are of the form demanded by the lemma. □

4.1. MOOTNESS. Not all properties impose distinctions on every class of grammars. For example, the property Un.o/X distinguishes one class of dense languages from another, but makes no distinction between types of sparse languages.

If Un.o/X = ⟨F.dom, A.dom⟩<> Ps, as assumed above, then Un.X contradicts Mult.sp and Un.o is entailed by Mult.sp, as shown in Lemma (27). In this case, the ineffectiveness of Un.o/X follows from the logic of its formulation, because only consistent conjunctions of values denote grammars, which are by definition non-empty sets of linear orders on CON$_S$ for some S.

We may also give the property a simpler formulation as F.dom <> Ps, omitting mention of A.dom, if we explicitly limit its scope to the grammars satisfying Mult.D. Under this approach, A&P (in prep.) describe the property Un.o/X as being *moot* with respect to Mult.sp. Nothing about the arguments above would change if we shifted to the scope-limited version. In nGX, there is no particularly strong reason to choose one over the other, but A&P show that mootness is a fundamental, ineradicable characteristic of typological structure in the general case, which is essential to the development of a property theory based on properties limited to the form A<>B, with appeal only to the dom and sub operators over constraint classes.

5. EXTENSIONAL TRAITS OF THE nGX OPTIMA

The results of §§1-4 classify the languages of nGX based on the property values that define their grammars. The property Un.o is given in its wide-scope version.

(32) nGX by Properties

Optima	Properties			
	Mult	FTyp	Pos	Un
Fo^k	sp	ia/tr	L	o
o^kF	sp	ia/tr	R	o
F^n(o)	D	ia/tr	L	o
(o)F^n	D	ia/tr	R	o
(U)F^n	D	ia/tr	L	X
F^n(U)	D	ia/tr	R	X

We now justify the classification.

(33) **Theorem**. **Optimal Outputs of nGX**. The optimal outputs of nGX of length greater than or equal to 2 syllables are drawn from the following patterns. Each, by GEN_{nGX}, is a single PrWd. Within each schema, F is uniformly iambic or uniformly trochaic, and $n \geq 1$, $k \geq 0$.

Mult/Un values	Pos.L	Pos.R
Mult.sp, Un.o	Fo^k	o^kF
Mult.D, Un.o	$\text{F}^n\text{(o)}$	(o)F^n
Mult.D, Un.X	(U)F^n	$\text{F}^n\text{(U)}$

Proof. From Proposition (16) we have that in any grammar satisfying Pos.L, optimal outputs take the form $[\text{(U)F}^n\text{o}^k]$ and in any grammar satisfying Pos.R, optimal outputs take the form $[\text{o}^k\text{F}^n\text{(U)}]$. By Proposition (11), the binary feet in such forms are either all iambic or all trochaic. According to Proposition (22), in any grammar satisfying Mult.sp, optimal outputs must be of the form $[\text{Fo}^k]$ or $[\text{o}^k\text{F}]$ with a single binary foot per word. From Proposition (25), in any grammar satisfying Mult.D, optimal outputs are restricted to the form $[\text{F}^n\text{(o)}]$ or $[\text{(U)F}^n]$ under Pos.L and $[\text{(o)F}^n]$ or $[\text{F}^n\text{(U)}]$ under Pos.R, $n \geq 1$. A grammar with the value Un.X has only completely parsed forms $[\text{(U)F}^n]$ or $[\text{F}^n\text{(U)}]$ by Proposition (29), and in any grammar with the value Un.o, there are no optima with unary feet, by Proposition (31), allowing only the forms $[\text{F}^n\text{(o)}]$ or $[\text{(o)F}^n]$. □

6. UNIVERSAL SUPPORTS FOR NGX

(34) **Theorem**. **Universal Supports for nGX**. Any collection of candidate sets that delivers the 12 grammars obtained from the properties in (4) is a universal support for nGX.

Proof. The properties in (4) entail the characterization of the optima asserted in Theorem (33). Each extensional description, when variables *n* and *k* are specified and when any parenthesized element is either included or omitted, determines the length of the described form in syllables, the location of its feet with respect to the syllable string and, when F is specified as iambic or trochaic, the location of the heads of the feet. By Remark (2), there can be only one form admitted by GEN_{nGX} that meets this description. Therefore, no further refinements of the grammars can be motivated. □

We may also now completely characterize the universal supports of nGX. The key observation is that a universal support must distinguish all the possible optima. The simplest case is when the optima manifest in a single candidate set.

(35) **Lemma. The Long Supports**. Any odd-length input of 5 or more syllables provides a universal support for nGX.

Proof. Consider an input of length $2m+1$, $m \geq 2$. Since it admits all parses defined by GEN_{nGX}, it admits exactly these from the table accompanying Theorem (33).

Mult	Un	Pos.L	Pos.R
sp	(o)	Fo^{2m-1}	o^{2m-1}F
D	o	F^m o	o F^m
D	X	U F^m	F^m U

Split each cell in two by allowing F to be uniformly iambic or uniformly trochaic in the formulae. These 12 forms are exactly the optimal outputs admitted by each of the grammars of nGX, as we have shown. Thus the candidate set for input of length $2m+1$, $m\geq 2$, contains 12 distinct optima, one for each grammar of nGX. □

(36) **Lemma**. **Failure of the Even**. No even length input provides a universal support for nGX.

Proof. The input of length 2 yields only the two optima shaped [F], by Remark (19). For the lengths $2m$, $m\geq 2$, we turn to Theorem (33), which gives a complete list of the shapes of all possible optima of nGX. No forms [F^no], [UF^n] [oF^n], [F^nU] can appear as parses of even lengths. The possibly optimal parses of even length 4 or greater are exhaustively those of the shapes [Fo^{2k}], [o^{2k}F], and [F^n], $n\geq 2$, $k\geq 0$. Among the dense, which fall under Mult.D, the longer even length inputs fail therefore to distinguish between Un.o and Un.X. □

We know from experiment that the support with inputs of length 3 and 4 syllables is universal, simply because it produces the 12 grammars. We may now generalize this observation to cover the collocation of the length 3 cset with any other cset with input of even length greater than 2.

(37) **Lemma**. The support with two inputs, one of length 3, the other of length $2m$, $m\geq 2$, is universal and minimal.

Proof. Length 3 has the optima [Fo], [oF], [UF], [FU], for F iambic and for F trochaic. Observe that the optimal structures [Fo^k] and [F^n(o)] are not distinguished at length 3, nor are [o^kF] and [(o)F^n]. This establishes that the input of length 3 does not provide a universal support. The structural types neutralized at length 3 are however distinguished at length $2m$, $m\geq 2$, where the optima [Fo^{2m-2}] instantiate [Fo^k], contrasting with the optima [F^m] of the shape [F^n(o)]; and the same mirror-imagewise for [o^{2m-2}F]. Here we tabulate the forms under Pos.L.

Mult	Un	3σ	$2m$ σ
sp	(o)	Fo	Fo^{2m-2}
D	o	Fo	F^m
D	X	UF	F^m

From Corollary (36), we know that the optima of length $2m$ are [F^m] as well as [Fo^{2k}], [o^{2k}F] for $k = m-1$, doubled by the iambic-trochaic distinction. From both lengths taken together, we may form pairs that distinguish all the grammars. Assuming Pos.L, these take the schematic shapes (Fo, Fo^{2m-2}), (Fo, F^m), and (UF, F^m), which may be expanded to instantiate the full range of Pos.L/R and FTyp.ia/tr values, yielding the 12 grammars. □

This establishes universality. Since neither length 3 nor length $2m$, $m \geq 2$, is universal by itself, we have minimality as well.

(38) **Theorem**. **Minimal Universal Supports for nGX**. The minimal universal supports for nGX are (1) the csets from an input of length 3 and an input of length $2m$, $m \geq 2$**,** and (2) any single cset from an input of odd length 5 syllables or greater. There are no others.

Proof. Lemma (37) establishes that the combination of csets from lengths 3 and $2m$, $m \geq 2$ is universal and minimal. Lemma (35) establishes the odd lengths greater than or equal to 5 syllables are each universal, with minimality following trivially. No even length can be universal by itself, by Lemma (36).

Any support Σ that contains an even length cset must therefore contain another cset of odd length. If the length of the odd member is greater than or equal to 5, then Σ is non-minimal. We now dispose of the shorter lengths. The 1 syllable cset has the same optimum in all grammars, and therefore belongs to no minimal support.

The 2-syllable input has only two optima, both shaped F, for iambic and trochaic. Every optimal output of length greater than 1 contains F, by Proposition (9). That F must be either iambic or trochaic, so that every such output determines the value of FTyp.ia/tr. The 2-syllable cset may therefore be removed from any universal support without compromising universality. We conclude that the 1 and 2 syllable csets belong to no minimal universal supports, and with that, we have covered all the cases. □

7. CONCLUSION

Along the route to characterizing the universal supports for nGX, we have established the shape of every optimum admitted by the system, and we have associated each optimum with the grammar that admits it. The essential move was to connect each property value with the extensional traits that it imposes. The resulting view of the system now lies well beyond what can be abstracted from observing the distribution of traits in a finite sample from nGX. We definitively *have* the typology in a way that sets the stage for wide-ranging analysis and comparison with other systems, abstract and concrete.

The method employed throughout is both simple and powerful. We find a minimal two-way contrast in forms and show that under certain assumptions about ranking —including, sometimes, none —one member of the pair is *better than* the other, in the technical sense used in the definition of optimality in OT. Because of the way candidates are ordered by OT grammars, it follows that the worse of the two can never be optimal in any grammar that meets the ranking assumptions. In this way, we have been able to project from local configurations to global patterns embracing entire forms, capitalizing on the order structure imposed by the theory.

We have used this tactic here to progressively narrow down the characterization of what can be optimal until only one candidate is left for each consistent collocation of intensional property values. Although we have sought completeness here, the method itself can be used more widely to gain partial knowledge of the extensional traits of a system's typology, and thus recommends itself to those who wish, for any reason, to make sense of the consequences of $\textsc{Con}_S$ for the range of candidates from $\textsc{Gen}_S$ that emerge as optima and the way they cluster in languages.

APPENDIX 1. SAMPLE OF THE EXTENSIONAL TYPOLOGY OF nGX: 2σ-5σ INPUTS

NB: All outputs for the 1σ input are U = -X-.

Language	**2s**	**3s**	**4s**	**5s**	**Footing**
sp.ia.L(.o)	-uX-	-uX-o-	-uX-o-o-	-uX-o-o-o-	Fo^{n}
sp.tr.L(.o)	-Xu-	-Xu-o-	-Xu-o-o-	-Xu-o-o-o-	
sp.ia.R(.o)	-uX-	-o-uX-	-o-o-uX-	-o-o-o-uX-	$\mathrm{o}^{n}\mathrm{F}$
sp.tr.R(.o)	-Xu-	-o-Xu-	-o-o-Xu-	-o-o-o-Xu-	
D.ia.L.o	-uX-	-uX-o-	-uX-uX-	-uX-uX-o-	$\mathrm{F}^{n}\mathrm{(o)}$
D.tr.L.o	-Xu-	-Xu-o-	-Xu-Xu-	-Xu-Xu-o-	
D.ia.R.o	-uX-	-o-uX-	-uX-uX-	-o-uX-uX-	$\mathrm{(o)F}^{n}$
D.tr.R.o	-Xu-	-o-Xu-	-Xu-Xu-	-o-Xu-Xu-	
D.ia.L.X	-uX-	-X-uX-	-uX-uX-	-X-uX-uX-	$\mathrm{(U)F}^{n}$
D.tr.L.X	-Xu-	-X-Xu-	-Xu-Xu-	-X-Xu-Xu-	
D.ia.R.X	-uX-	-uX-X-	-uX-uX-	-uX-uX-X-	$\mathrm{F}^{n}\mathrm{(U)}$
D.tr.R.X	-Xu-	-Xu-X-	-Xu-Xu-	-Xu-Xu-X-	

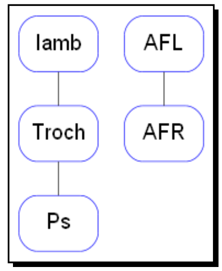
Appendix 2. Grammars of nGX given as SKBs and Hasse diagrams

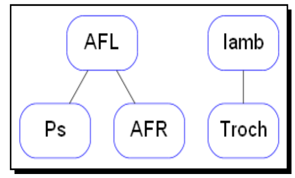
Computed in OTWorkplace (Prince, Tesar, & Merchant 2015).

Sparse. Outputs = [F,o^n]

sp.ia.L

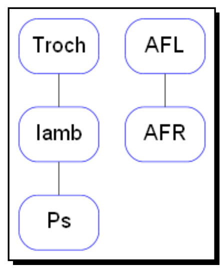
4:AFL	5:AFR	2:Iamb	3:Troch	1:Ps
W	**L**			
		W	**L**	
W			**W**	**L**

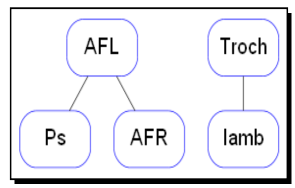




sp.tr.L

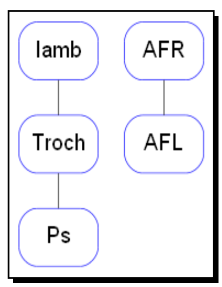
4:AFL	5:AFR	3:Troch	2:Iamb	1:Ps
W	**L**			
		W	**L**	
W			**W**	**L**

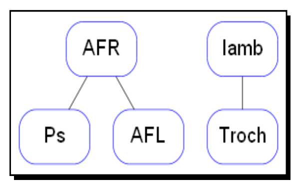




sp.ia.R

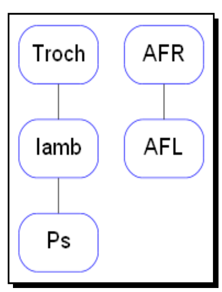
5:AFR	4:AFL	2:Iamb	3:Troch	1:Ps
W	**L**			
		W	**L**	
W			**W**	**L**

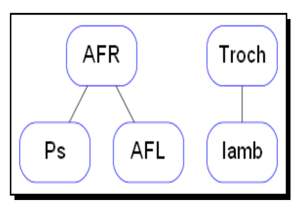




sp.tr.R

5:AFR	4:AFL	3:Troch	2:Iamb	1:Ps
W	**L**			
		W	**L**	
W			**W**	**L**

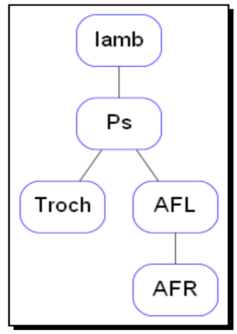




D.o = ‘WEAKLY DENSE’. OUTPUTS = [F^n,o]

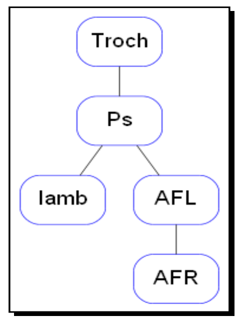
D.ia.L.o

2:Iamb	1:Ps	3:Troch	4:AFL	5:AFR
W	**L**			
	W	**L**	**L**	
			W	**L**



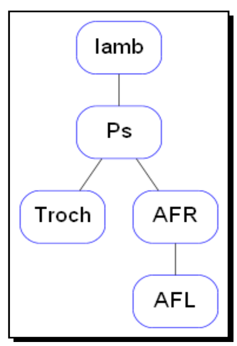
D.tr.L.o

3:Troch	1:Ps	2:Iamb	4:AFL	5:AFR
W	**L**			
	W	**L**	**L**	
			W	**L**



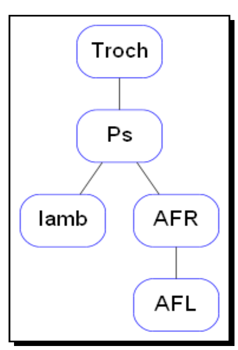
D.ia.R.o

2:Iamb	1:Ps	3:Troch	5:AFR	4:AFL
W	**L**			
	W	**L**	**L**	
			W	**L**



D.tr.R.o

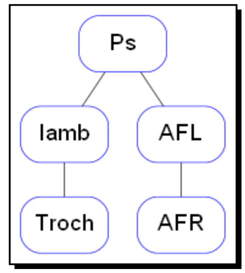
3:Troch	1:Ps	2:Iamb	5:AFR	4:AFL
W	**L**			
	W	**L**	**L**	
			W	**L**



D.X = ‘STRONGLY DENSE’. OUTPUTS = [F^n,U]

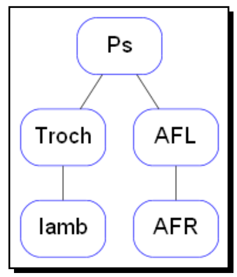
D.ia.L.X

1:Ps	2:Iamb	4:AFL	3:Troch	5:AFR
W	**L**	**L**		
	W		**L**	
		W		**L**



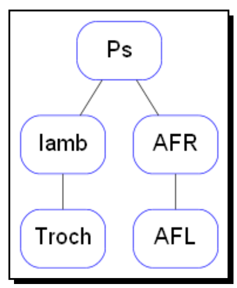
D.tr.L.X

1:Ps	3:Troch	4:AFL	2:Iamb	5:AFR
W	**L**	**L**		
	W		**L**	
		W		**L**



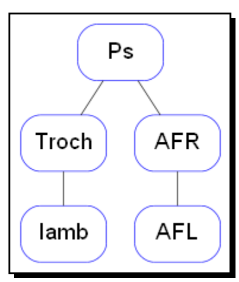
D.ia.R.X

1:Ps	2:Iamb	5:AFR	3:Troch	4:AFL
W	**L**	**L**		
	W		**L**	
		W		**L**



D.tr.R.X

1:Ps	3:Troch	5:AFR	2:Iamb	4:AFL
W	**L**	**L**		
	W		**L**	
		W		**L**



APPENDIX 3. PROPERTY SPECIFICATIONS IN THE IAMBIC LEFT QUADRANT OF nGX

(i) sp.ia.L.o "Sparse Iambic Left"

Property	Value	ERC
Mult	sp	L.eW.We
Ftyp	ia	e.WL.ee
Pos	L	e.ee.WL
Un	o	L.We.We

(ii) D.ia.L.o "Weakly Dense Iambic Left"

Property	Value	ERC
Mult	D	W.eL.Le
Ftyp	ia	e.WL.ee
Pos	L	e.ee.WL
Un	o	L.We.We

(ii) D.ia.L.X "Strongly Dense Iambic Left"

Property	Value	ERC
Mult	D	W.eL.Le
Ftyp	ia	e.WL.ee
Pos	L	e.ee.WL
Un	X	W.Le.Le

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