What is OT?

What is OT?

Wir kannten nicht sein unerhörtes Haupt
darin die Augenäpfel reiften. Aber …

What is OT?

Neo: I know kung fu.
Morpheus: [*eyeing him, hand on chin*] Show me.

What is OT?

A concise overview

Alan Prince

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Tableaux and diagrams constructed and calculations conducted in OTWorkplace (Prince, Merchant, and Tesar 2007-2017).

January 31, 2016

Goals

The aim is to provide what Leonard Susskind in another domain has ambitiously described as ‘the theoretical minimum’ — here, for understanding and working in or on OT. The conceptual perspective is that of the original work in the area, as further developed and clarified in the late 20th and early 21st century.

Presentation is relatively concise, in the interests of constructing an unobstructed view of OT as a theory of choice and the way that its premises guide analysis.

What is OT?

A concise overview

Alan Prince

- The System
- The Objects of OT
- Optimality
- Analysis

What is OT?

on a postcard

The System. S = $\langle$S.GEN, S.CON$\rangle$.

The Objects. Language, grammar, typology.

Optimality. Better on a constraint,
Better on a hierarchy of constraints,
Best.

Analysis. S $\Rightarrow$T $\Rightarrow$G, an analysis if it exists under S.

- The System
- The Objects of OT
- Optimality
- Analysis

Working within the System

OT is defined within a ***system***:

- a fully-specified formal entity
  about which true and false things may be said.

- not chunks raggedly extracted from a looming, unknown whole (“Human Language”),
  about which hedged, vague things may be said
  or grand things imagined.

What is True and What is Not

In approaching reality, we proceed from system to system
navigating in the direction of universal grammar.
- So we always know what we're talking about.
- Or stand a chance of figuring it out.

A given *system* may be studied to shed light on the theory.
- Not because it tightly models some presumed facts.

Overfitting is the enemy of understanding.

Why the System

The need devolves from the definition of *optimality.*

An optimal form is better than all distinct candidates. *All*!

- The set of competitors must be defined.

Comparison polls the judgment of all constraints. *All*!

- The entire constraint set must be specified.

Outside the bounds of the *system*, *optimality* loses meaning.

$S = \langle S.\textsc{Gen}, S.\textsc{Con} \rangle$

S.GEN defines the candidates and candidate sets of S.

A definition need not be a procedure:

its role is to delimit, unambiguously.

S.CON defines the constraints of S.

A constraint is a function from candidates to the non-negative integers 0,1,…

It associates each *candidate* from S.GEN with a penalty.

GEN and CON from Prince & Smolensky 1993/2004 et seq.

Role of the System

Analysis. Every *OT analysis* occurs within a system S,
and rests on the definitions of S.GEN and S.CON.
Because an 'analysis' is a grammar or set of grammars
admitted by S.

No Exit. This is true whether or not these are defined explicitly.
Because the force of logic is inescapable.

Same, same. Much the same will be true, in reality, of rational
discourse within other theories when clearly defined.
- OT, taken seriously, allows very little wiggle room.

- The System
- **The Objects of OT**
- Optimality
- Analysis

The Objects of OT

1. **Language**

The *candidates* **optimal** under a given constraint hierarchy.

2. **Grammar**

The *set of all hierarchies* that yield the same language.

- a *hierarchy* being a linear order on the constraints of S.CON

3. **Typology**

Extensional. The *languages* of S.

Intensional. The *grammars* of S.

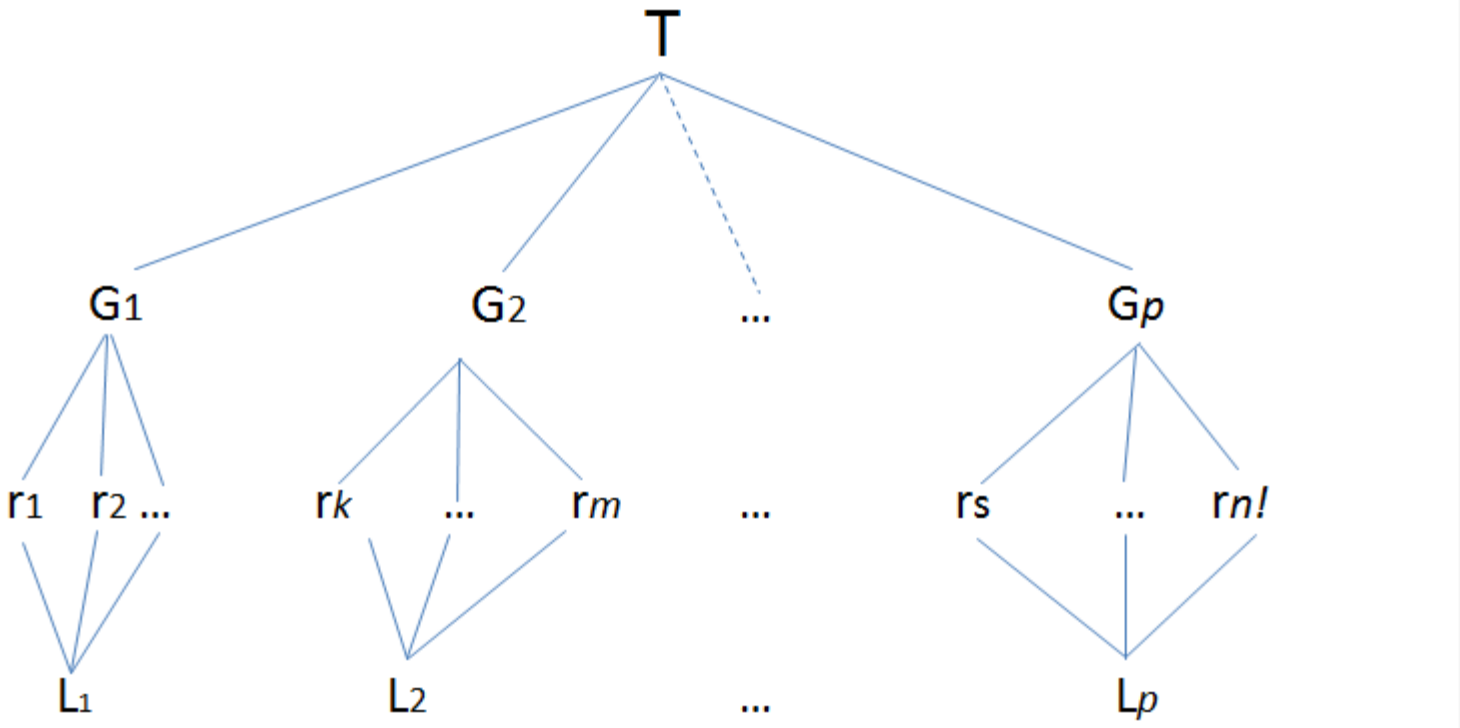
To understand S,

we must understand the typology of S.

To understand the typology of S,

we must understand its languages and their grammars.

The Objects of OT



A typology consists of grammars;
a grammar, of all the hierarchies that deliver the same optima.

Why *all*?

Think of a grammar not just as a collection of hierarchies
but as the set of requirements on specific constraints
necessary & sufficient to choose its optimal candidates.
- e.g. Iamb >> Trochee, f.dep & f.max >> m.Ons, and so on.

These give the essential connection between theory and data:
the linguistically significant restrictions
within the theory that the data imposes.

And these conditions collectively denote *every* hierarchy
that yields the optima. So: a grammar holds them all.

A Prosodic *System*

nGX

nGX.GEN: defining Candidate, Candidate set, Input, Output.

Candidate: A pair (Input, Output).

Candidate set (*cset*): All such pairs with same Input.

Input: A string of syllables, taken as atomic units.

Output for a given input: All parses of the input into a single Prosodic Word; no deletions or insertions.

Analyzed in Alber & Prince, 2015, 2017, in prep., and Alber, DelBusso, & Prince 2016.

nGX.GEN: the structures

Categories

A **PrWd** consists of feet and syllables.

A **Foot** consists of syllables.

A **Syllable** is atomic.

Arrangements

- Constituents neither overlap nor recurse.
- A Foot may contain 1 or 2 syllables
- Every Foot has a unique *head* syllable
- Syllables may be parsed into Feet or not, freely.
- Every Prosodic Word contains at least one foot.

Spelling nGX.GEN

nGX assumes a familiar constituent structure for prosody.

For convenience, we allude to its elements as follows:

X	head of foot
u	nonhead of foot
o	syllable not parsed into foot
-	edge of prosodic unit

Thus:

-Xu-	trochaic foot: *head initial*
-uX-	iambic foot: *head final*
-X-	monosyllabic foot: *head initial and head final*
-o-	unfooted syllable

nGX.CON

Name	***Def.***	***Verbose***: For each candidate, the constraint returns *the number of matches in Output* to:
1**. Parse-s**	*o	a syllable not belonging to a foot.
2. **Iamb**	*-X	a head-initial foot: -Xu- , -X-
3. **Troch**	*X-	a head-final foot: -uX- , -X-
4. **AFL**	*(σ,ft): σ…ft	each pair (syll , foot), where σ precedes ft.
5. **AFR**	*(σ,ft): ft…σ	each pair (syll , foot) , where ft precedes σ.

- *P takes a candidate as argument, returns the number of matches in it to pattern P.
- Defns. of AFL ("All Feet Left"), AFR ("All Feet Right") simplified from Hyde 2012.
- Defns. of Iamb, Troch are 'new' in that they penalize, rather than accept, -X-.

nGX Sampled

Violation Tableaux — VTs.

2s and 3s csets complete:
all admitted candidates.

Plus beginning of 4s
continuing downward…
to list the 32 more 4s cands.

input	output	opt	P-s	Iamb	Troch	AFL	AFR
oo	-uX-		0	0	1	0	0
	-o-X-		1	1	1	1	0
	-Xu-		0	1	0	0	0
	-X-o-		1	1	1	0	1
	-X-X-		0	2	2	1	1
ooo	-o-uX-		1	0	1	1	0
	-o-o-X-		2	1	1	2	0
	-uX-o-		1	0	1	0	1
	-uX-X-		0	1	2	2	1
	-o-Xu-		1	1	0	1	0
	-o-X-o-		2	1	1	1	1
	-o-X-X-		1	2	2	3	1
	-Xu-o-		1	1	0	0	1
	-Xu-X-		0	2	1	2	1
	-X-o-o-		2	1	1	0	2
	-X-uX-		0	1	2	1	2
	-X-o-X-		1	2	2	2	2
	-X-Xu-		0	2	1	1	2
	-X-X-o-		1	2	2	1	3
	-X-X-X-		0	3	3	3	3
oooo	-o-o-uX-		2	0	1	2	0
	-o-o-o-X-		3	1	1	3	0
	-o-uX-o-		2	0	1	1	1
	-o-uX-X-		1	1	2	4	1
	-o-o-Xu-		2	1	0	2	0
	-o-o-X-o-		3	1	1	2	1
	-o-o-X-X-		2	2	2	5	1
	-uX-o-o-		2	0	1	0	2
	-uX-uX-		0	0	2	2	2
	-uX-o-X-		1	1	2	3	2
	-uX-Xu-		0	1	1	2	2
	…		…	…	…	…	…

nGX is so named because
- ***n***ew defn. of Iamb/Troch used
- **G**eneralized Alignment positions feet
- **X** occurs at least once in every word

What is nGX about?

nGX deals with aspects of stress prosody that depend on
- grouping into feet
- headedness of feet
- foot population of word

nGX abstracts away from
- distinctions in prominence among foot heads
- effects of the internal composition of syllables

The excluded traits often function independently of those represented, making this a mild abstraction.

- Mildness I: *Many patterns arrange feet without reference to main stress.*
- Mildness II: *Every QS system has a QI system inside it.*
- Far more drastic abstractions are undoubtedly necessary for theoretical advance.

- The System
- The Objects of OT
- Optimality
- Analysis

Optimality: both sides

We look at optimality in two ways, to answer different questions.

1. **Filtration**. Suppose we have a hierarchy in hand.
   What candidate or candidates does it select as optimal?

2. **Comparison**. Suppose we have a candidate desired optimal.
   What ranking requirements do those hierarchies meet
   that choose it as optimal, better than all competitors?

A 'hierarchy' or 'ranking' is a linear order on the constraint set S.CON.

Optimality by Mass Filtration

Take the Best, Ignore the Rest */verbose=on*

1. Filtration **by a single constraint C** of a set of candidates K.
   C[K] is the best of K as judged by C: smallest penalty.

2. Filtration **by a hierarchy of constraints H** = $C_1 \gg C_2 \gg \ldots \gg C_n$.
   - Take the best on C_1 = C_1[K] — *the best*
   - Filter this with C_2 to get $C_2[C_1[K]]$ — *the best of the best*
   - Iterate until you've gone through all of H. —*... of the best*

3. **Optimality**
   A candidate is *optimal*
   iff it's among the results of filtering by an entire hierarchy.

"Take the best…." from Gigerenzer & Goldstein 1996.

Optimality by Mass Filtration

Take the Best, Ignore the Rest */verbose=off*

1. Filtration of a set of candidates K by a single constraint C.

   $C[K] = \{\mathbf{q} \in K \mid C(\mathbf{q})$ is **minimal**: for all $z \in K$, $C(\mathbf{q}) \leq C(\mathbf{z})\}$

   - $C(\mathbf{q})$ is the integer value assigned by C to **q**.
   - C[K] is the *best* of K, as judged by C.

2. Filtration by a hierarchy H = CP (P a sequence of constraints)

   $H[K] = CP[K] = P[C[K]]$ (recursive formulation)

3. **Optimality**. For $H = C_1 \gg C_2 \gg \ldots \gg C_n$, $C_k \in$ S.CON

   $\mathbf{q} \in K$ is optimal on H iff $\mathbf{q} \in H[K]$**.**

VT with columns in ranking order

nGX.5s				*o	*-X	*X-	func:AFL	func:AFR
Cand#	input	output	opt	1:P-s	2:lamb	3:Troch	4:AFL	5:AFR
1	5s	-X-uX-uX-		0	1	3	4	6
2		-uX-uX-X-		0	1	3	6	4
3		-X-Xu-Xu-		0	3	1	4	6
4		-Xu-Xu-X-		0	3	1	6	4
5		-uX-uX-o-		1	0	2	2	4
6		-o-uX-uX-		1	0	2	4	2
7		-Xu-Xu-o-		1	2	0	2	4
8		-o-Xu-Xu-		1	2	0	4	2
9		-uX-o-o-o-		3	0	1	0	3
10		-o-o-o-uX-		3	0	1	3	0
11		-Xu-o-o-o-		3	1	0	0	3
12		-o-o-o-Xu-		3	1	0	3	0

A Filtration

nGX.5s				*o	*-X	*X-	func:AFL	func:AFR
Cand#	input	output	opt	1:P-s	2:lamb	3:Troch	4:AFL	5:AFR
1	5s	-X-uX-uX-		**0**	1	3	4	6
2		-uX-uX-X-		**0**	1	3	6	4
3		-X-Xu-Xu-		**0**	3	1	4	6
4		-Xu-Xu-X-		**0**	3	1	6	4
5		-uX-uX-o-		1	0	2	2	4
6		-o-uX-uX-		1	0	2	4	2
7		-Xu-Xu-o-		1	2	0	2	4
8		-o-Xu-Xu-		1	2	0	4	2
9		-uX-o-o-o-		3	0	1	0	3
10		-o-o-o-uX-		3	0	1	3	0
11		-Xu-o-o-o-		3	1	0	0	3
12		-o-o-o-Xu-		3	1	0	3	0

A Filtration

nGX.5s				*o	*-X	*X-	func:AFL	func:AFR
Cand#	input	output	opt	1:P-s	2:lamb	3:Troch	4:AFL	5:AFR
1	5s	-X-uX-uX-		0	**1**	3	4	6
2		-uX-uX-X-		0	**1**	3	6	4
3		-X-Xu-Xu-		0	3	1	4	6
4		-Xu-Xu-X-		0	3	1	6	4
5		-uX-uX-o-		1	0	2	2	4
6		-o-uX-uX-		1	0	2	4	2
7		-Xu-Xu-o-		1	2	0	2	4
8		-o-Xu-Xu-		1	2	0	4	2
9		-uX-o-o-o-		3	0	1	0	3
10		-o-o-o-uX-		3	0	1	3	0
11		-Xu-o-o-o-		3	1	0	0	3
12		-o-o-o-Xu-		3	1	0	3	0

A Filtration

nGX.5s				*o	*-X	*X-	func:AFL	func:AFR
Cand#	input	output	opt	1:P-s	2:lamb	3:Troch	4:AFL	5:AFR
1	5s	-X-uX-uX-		0	1	**3**	4	6
2		-uX-uX-X-		0	1	**3**	6	4
3		-X-Xu-Xu-		0	3	1	4	6
4		-Xu-Xu-X-		0	3	1	6	4
5		-uX-uX-o-		1	0	2	2	4
6		-o-uX-uX-		1	0	2	4	2
7		-Xu-Xu-o-		1	2	0	2	4
8		-o-Xu-Xu-		1	2	0	4	2
9		-uX-o-o-o-		3	0	1	0	3
10		-o-o-o-uX-		3	0	1	3	0
11		-Xu-o-o-o-		3	1	0	0	3
12		-o-o-o-Xu-		3	1	0	3	0

A Filtration

nGX.5s				*o	*-X	*X-	func:AFL	func:AFR
Cand#	input	output	opt	1:P-s	2:lamb	3:Troch	4:AFL	5:AFR
1	5s	-X-uX-uX-		0	1	3	**4**	6
2		-uX-uX-X-		0	1	3	6	4
3		-X-Xu-Xu-		0	3	1	4	6
4		-Xu-Xu-X-		0	3	1	6	4
5		-uX-uX-o-		1	0	2	2	4
6		-o-uX-uX-		1	0	2	4	2
7		-Xu-Xu-o-		1	2	0	2	4
8		-o-Xu-Xu-		1	2	0	4	2
9		-uX-o-o-o-		3	0	1	0	3
10		-o-o-o-uX-		3	0	1	3	0
11		-Xu-o-o-o-		3	1	0	0	3
12		-o-o-o-Xu-		3	1	0	3	0

A Filtration

nGX.5s				*o	*-X	*X-	func:AFL	func:AFR
Cand#	input	output	opt	1:P-s	2:lamb	3:Troch	4:AFL	5:AFR
1	5s	-X-uX-uX-	***OPT!***	0	1	3	4	**6**
2		-uX-uX-X-		0	1	3	6	4
3		-X-Xu-Xu-		0	3	1	4	6
4		-Xu-Xu-X-		0	3	1	6	4
5		-uX-uX-o-		1	0	2	2	4
6		-o-uX-uX-		1	0	2	4	2
7		-Xu-Xu-o-		1	2	0	2	4
8		-o-Xu-Xu-		1	2	0	4	2
9		-uX-o-o-o-		3	0	1	0	3
10		-o-o-o-uX-		3	0	1	3	0
11		-Xu-o-o-o-		3	1	0	0	3
12		-o-o-o-Xu-		3	1	0	3	0

What we have got

Why are we sure that we've got the *optimum*?

Shouldn't we filter the entire set of 119 five-syllable forms?

- To be *optimal* is to be the best of *all*.

Here we filter only those 12 forms that are *possible optima*.
Some ranking exists that makes each of them optimal.

But the other 107 are never optimal:
under every ranking, they lose to something else.
They are said to be *harmonically bounded*.

They never affect the choice of optimum
nor the rankings that choose it.

On harmonic bounding, see Prince & Smolensky 1993/2004; Samek-Lodovici & Prince 1999.

The Selection Problem, Solved

The filtration view works splendidly for us when we have the hierarchy H in hand, as in the case just reviewed.

Having fixed S.GEN, S.CON, and H, we can find the optima of H.

This is the *selection problem*: what does H select from K?

We can solve it because we know

- All the candidates and what they compete with
- All the constraints
- The definition of optimality
- The particular hierarchy we want to filter with.

The Other Way Round

But the usual problem taken on by the analyst is quite different.

With observations, generalizations and representations in mind,
a set of desired *optima* is identified.

We know S.GEN & S.CON and we want certain optima to emerge.

- The ranking problem:
  What *hierarchies* select the desired optima?

The Ranking Problem, Solved

The ranking problem finds its resolution in the following fact:

Veridical ranking information can be obtained
by comparing 2 candidates over the entirety of S.CON,
with one asserted to be optimal.

This emerges from a deep feature of OT:
the independence of irrelevant alternatives.

Choice between candidates **q** and **z** depends only on **q** and **z**,
not on how any other choices are made.
- But knowledge of *all* constraints is still required.

To filter is to compare

an overview

OT Filtration recognizes just 3 relations between candidates:
Better than, Worse than, The Same as.

Suppose we filter the set with just the 2 candidates {**q**, **z**}.

- If **q** stays and **z** goes, then **q** is *better than* **z**.
- In the same situation, **z** is *worse than* **q**.
- If both survive, the **q** is *the same as* **z**.

□ By this, **q** will oust **z** from {**q**,**z**} when filtered by a hierarchy H iff
some constraint preferring **q** dominates *any & all* preferring **z**.
- Because the first distinguishing constraint in the order decides.

One against all

Working pairwise, we can exhaust the entire set of competitors

$\mathbf{q} \sim \mathbf{z}_1$
$\mathbf{q} \sim \mathbf{z}_2$
…
$\mathbf{q} \sim \mathbf{z}_n$

Each face-off provokes every constraint in S.CON to issue
better-than/worse-than/ same-as judgements

The totality of such competitions delimits the exact conditions
required for the optimality of **q**:

□ For each $\mathbf{z}_k$, some constraint favoring **q** over $\mathbf{z}_k$
must dominate every constraints favoring $\mathbf{z}_k$ over **q**.

Optimality by Pairwise Comparison

/verbose=on

1. **Better than** on a constraint

q is *better than **z*** on C

iff C assigns a smaller penalty to **q** than to **z**.

2. **Better than** on a hierarchy H

q is *better than* **z** *on H*

iff **q** is *better than* **z** on

the highest ranked *constraint* in H

that assigns different penalties to **q** and **z**.

3. **Optimal**

q is optimal in K on H

iff **q** is *better on H* than ***all*** (violation-distinct) competitors.

The caveat 'violation-distinct' excludes from competition with **q** any candidate that has the same violation profile as **q**, which OT see as having exactly the same properties as **q**.

Optimality by Pairwise Comparison

/verbose=off

1. **Better than** on a constraint C. For candidates **q**,**z**

q is *better than* **z** *on* C iff C(**q**) < C(**z**).

2. **Better than** on a hierarchy H = CP

q is *better than* **z** *on* H = CP iff

- **q** is better than **z** on C, or

if C(**q**) = C(**z**) (*i.e*. **z** is *not* better than **q** on C)

then **q** is better than **z** on the hierarchy P.

3. **Optimal** in K on H

q is optimal in K on H

iff **q** better than **z** on H for *all* (violation-distinct) **z** in K.

Nothing new under the Sun

The definition of *optimality* from pairwise comparison is equivalent to the mass filtration definition.

The same candidates are optimal under both.

The two ways of thinking comport with different situations.

Outside of doctrine-driven discourse, it's good not bad to command several ways of thinking about things.

Comparing a Pair

Suppose we want **q** better than **z**.

- *Which hierarchies make this happen*?

We construct the **Elementary Ranking Condition (ERC) q$\sim$z,**

collecting the judgment of *every* constraint C$\in$S.CON

on the relation between **q** and **z**.

From **C**, we build a new kind of function **dC**,

which returns the effective ***d****ifference* between **q** and **z**.

Win, Lose, or Draw

Let us define **dC**, a function of pairs, from **C**$\in$S.CON.

1) **dC(q,z)** = **W** when C{q,z} = {q}, *i.e.* C(q)<C(z)
'q is better than z on C'

2) **dC(q,z)** = ***e*** when C{q,z} = {q,z}, *i.e.* C(q) = C(z)
'q is the same as z on C'

3) **dC(q,z)** = **L** when C{q,z} = {z}, *i.e.* C(z) < C(q)
'z is better than q on C'

By the Bootstraps

C and dC are different functions: dC is *derived from* C$\in$S.CON.

C: $\text{CAND}_S \rightarrow \{0, 1, 2,\ldots\}$ CAND_S = candidates of S.

dC: $\text{CAND}_S \times \text{CAND}_S \rightarrow \{W,L,e\}$

- C maps from candidates to the non-negative integers.
- dC maps from pairs of candidates to a 3-element set.

They provide different information.

They do *not* display the same information in a different 'format'.

We observe a notational distinction here for purposes of clarity.

Though not customary, this disables the murky practice of displaying both kinds on information in the same tableau.

What the ERC says

The ERC q∼z is the collection of evaluations **dC(q,z)** for all C in S.CON.

Conventionally presented as list ('vector')with some arbitrary order on the constraints.

ERC	dIamb	dAFR	dTroch	dP-s	dAFL
q ∼ z	*e*	**W**	**W**	**L**	*e*

The ERC vector **q∼z** tells us this:

In every hierarchy in which **q** is *better than* **z**

either **AFR (W)** *or* **Troch (W)** dominates **P-s (L).**

Why?

If P-s were encountered before AFR and Troch in any filtration,

z would be chosen from {**q,z**}, and thus be better than q.

Obtaining **q**

Suppose we want **q** *better than* **z**.

What hierarchies **H** make this happen?

- We examine the **ERC q∼z** to find out.

Elementary Ranking Condition. Some W dominates all L's.

This is what we learn from comparing with a desired optimum.

Grammar. A *grammar* is a set of ERCs that collectively tells us how to make **q** better than **all** violation-distinct competitors **z**.

The **ERC** is the essential quantum of ranking information from which grammars are built.

Winning is Everything

nGX.5s								
Cand#	input	output	opt	P-s	lamb	Troch	AFL	AFR
1	5s	-X-uX-uX-	**WIN!**	0	1	3	4	6
2		-uX-uX-X-		0	1	3	6	4
3		-X-Xu-Xu-		0	3	1	4	6
4		-Xu-Xu-X-		0	3	1	6	4
5		-uX-uX-o-		1	0	2	2	4
6		-o-uX-uX-		1	0	2	4	2
7		-Xu-Xu-o-		1	2	0	2	4
8		-o-Xu-Xu-		1	2	0	4	2
9		-uX-o-o-o-		3	0	1	0	3
10		-o-o-o-uX-		3	0	1	3	0
11		-Xu-o-o-o-		3	1	0	0	3
12		-o-o-o-Xu-		3	1	0	3	0

Divide and Conquer

nGX.5s								
Erc	Input	Winner	Loser	dP-s	dlamb	dAFL	dTroch	dAFR
1~8	5s	-X-uX-uX-	-o-Xu-Xu-	W	W		L	L
1~7	5s	-X-uX-uX-	-Xu-Xu-o-	W	W	L	L	L
1~11	5s	-X-uX-uX-	-Xu-o-o-o-	W		L	L	L
1~12	5s	-X-uX-uX-	-o-o-o-Xu-	W		L	L	L
1~6	5s	-X-uX-uX-	-o-uX-uX-	W	L		L	L
1~5	5s	-X-uX-uX-	-uX-uX-o-	W	L	L	L	L
1~9	5s	-X-uX-uX-	-uX-o-o-o-	W	L	L	L	L
1~10	5s	-X-uX-uX-	-o-o-o-uX-	W	L	L	L	L
1~4	5s	-X-uX-uX-	-Xu-Xu-X-		W	W	L	L
1~3	5s	-X-uX-uX-	-X-Xu-Xu-		W		L	
1~2	5s	-X-uX-uX-	-uX-uX-X-			W		L

P-s ≫ {lamb, Troch, AFL, AFR}

nGX.5s								
Erc	Input	Winner	Loser	dP-s	dlamb	dAFL	dTroch	dAFR
1~8	5s	-X-uX-uX-	-o-Xu-Xu-	W	W		L	L
1~7	5s	-X-uX-uX-	-Xu-Xu-o-	W	W	L	L	L
1~11	5s	-X-uX-uX-	-Xu-o-o-o-	W		L	L	L
1~12	5s	-X-uX-uX-	-o-o-o-Xu-	W		L	L	L
1~6	5s	-X-uX-uX-	-o-uX-uX-	W	L		L	L
1~5	5s	-X-uX-uX-	-uX-uX-o-	W	L	L	L	L
1~9	5s	-X-uX-uX-	-uX-o-o-o-	W	L	L	L	L
1~10	**5s**	**-X-uX-uX-**	**-o-o-o-uX-**	**W**	**L**	**L**	**L**	**L**
1~4	5s	-X-uX-uX-	-Xu-Xu-X-		W	W	L	L
1~3	5s	-X-uX-uX-	-X-Xu-Xu-		W		L	
1~2	5s	-X-uX-uX-	-uX-uX-X-			W		L

Iamb ≫ Troch

nGX.5s								
Erc	Input	Winner	Loser	dP-s	dIamb	dAFL	dTroch	dAFR
1~8	5s	-X-uX-uX-	-o-Xu-Xu-	W	W		L	L
1~7	5s	-X-uX-uX-	-Xu-Xu-o-	W	W	L	L	L
1~11	5s	-X-uX-uX-	-Xu-o-o-o-	W		L	L	L
1~12	5s	-X-uX-uX-	-o-o-o-Xu-	W		L	L	L
1~6	5s	-X-uX-uX-	-o-uX-uX-	W	L		L	L
1~5	5s	-X-uX-uX-	-uX-uX-o-	W	L	L	L	L
1~9	5s	-X-uX-uX-	-uX-o-o-o-	W	L	L	L	L
1~10	5s	-X-uX-uX-	-o-o-o-uX-	W	L	L	L	L
1~4	5s	-X-uX-uX-	-Xu-Xu-X-		W	W	L	L
1~3	**5s**	**-X-uX-uX-**	**-X-Xu-Xu-**		**W**		**L**	
1~2	5s	-X-uX-uX-	-uX-uX-X-			W		L

AFL ≫ AFR

nGX.5s								
Erc	Input	Winner	Loser	dP-s	dlamb	dAFL	dTroch	dAFR
1~8	5s	-X-uX-uX-	-o-Xu-Xu-	W	W		L	L
1~7	5s	-X-uX-uX-	-Xu-Xu-o-	W	W	L	L	L
1~11	5s	-X-uX-uX-	-Xu-o-o-o-	W		L	L	L
1~12	5s	-X-uX-uX-	-o-o-o-Xu-	W		L	L	L
1~6	5s	-X-uX-uX-	-o-uX-uX-	W	L		L	L
1~5	5s	-X-uX-uX-	-uX-uX-o-	W	L	L	L	L
1~9	5s	-X-uX-uX-	-uX-o-o-o-	W	L	L	L	L
1~10	5s	-X-uX-uX-	-o-o-o-uX-	W	L	L	L	L
1~4	5s	-X-uX-uX-	-Xu-Xu-X-		W	W	L	L
1~3	5s	-X-uX-uX-	-X-Xu-Xu-		W		L	
1~2	5s	-X-uX-uX-	-uX-uX-X-			W		L

All that it takes

nGX.5s								
Erc	Input	Winner	Loser	dP-s	dlamb	dAFL	dTroch	dAFR
1~8	5s	-X-uX-uX-	-o-Xu-Xu-	W	W		L	L
1~7	5s	-X-uX-uX-	-Xu-Xu-o-	W	W	L	L	L
1~11	5s	-X-uX-uX-	-Xu-o-o-o-	W		L	L	L
1~12	5s	-X-uX-uX-	-o-o-o-Xu-	W		L	L	L
1~6	5s	-X-uX-uX-	-o-uX-uX-	W	L		L	L
1~5	5s	-X-uX-uX-	-uX-uX-o-	W	L	L	L	L
1~9	5s	-X-uX-uX-	-uX-o-o-o-	W	L	L	L	L
1~10	5s	-X-uX-uX-	-o-o-o-uX-	**W**	**L**	**L**	**L**	**L**
1~4	5s	-X-uX-uX-	-Xu-Xu-X-		W	W	L	L
1~3	5s	-X-uX-uX-	-X-Xu-Xu-		**W**		**L**	
1~2	5s	-X-uX-uX-	-uX-uX-X-			**W**		**L**

Follow the Logic

An ERC is a logical expression:

Some W dominates all L's.

W_j *or* …*or* W_m >> L_k *and* … *and* L_n

A logic of entailment is associated with expressions of this type.

By this logic, we can eliminate all redundancy from an ERC set.

We may even be able to simplify it beyond what we are given.

This can be done algorithmically.

Follow the Propositional Logic

ERC 1∼10 **[W.LL.LL]** says:

(1) **P-s** dominates all of {Iamb, Troch, AFL, AFR}

ERC 1∼7 **[W.WL.LL]** says:

(2) **P-s *or* Iamb** dominates all of {Troch, AFL, AFR}

BUT

If (1) '**P-s** dominates everything else' *is true,*
then (2) '**P-s *or* Iamb** dominates various other constraints' *is also true*.

Statement (2) is redundant and adds nothing to (1).

We may *omit it* from a defining collection of ERCs: a grammar.

Follow the ERC Logic

To reduce an ERC set to its essentials, we must also take account of the order properties of constraint hierarchies.

'$\gg$' is a strict order: asymmetric, irreflexive, transitive.

This reduction can be carried out by simple operations on W,L,e values, with no reference to $\wedge$,$\vee$, $\gg$ and their formal properties.

The useful ERC logic operation of combination is *fusion*. It gives us

[*e*.W*e*.L*e*] ◦ [*e*.*e*W.*e*L] = *e*.WW.LL i.e. [1~3] ◦[1~2] = [1~4]

allowing us to eliminate 1~4 in favor of {1~2, 1~3}.

- Fusion: L◦X = X, *e*◦X=X, W◦W=W. See e.g. Prince 2002, 2009.

- Fusion of the cited ERCs encapsulates in one stroke the following argument:
  - in any given hierarchy, one of the L's must dominate the other (strictness).
  - one of the W's must dominate it (from 1~2 and 1~3).
  - Therefore, one of the W's dominates both L's (transitivity).

A grammar

The ERC expresses the *kind* of ranking relations that data yields.

It can be used to represent those relations *per se*,
regardless of where they come from.

This ERC set represents our findings:

P-s	Iamb	AFL	Troch	AFR
W	L	L	L	L
	W		L	
		W		L

This is the MIB, the 'Most Informative Basis', of the grammar. See Brasoveanu & Prince 2005/11.

A grammar

Further calculations can be made.

If we remove all L's derivable from other ERCs in the set,
namely those that follow from the transitivity of ranking,

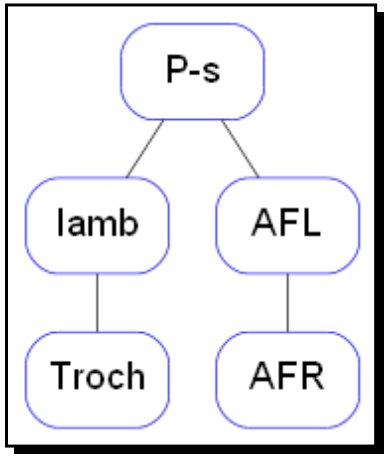
We arrive at a sparser representation:

P-s	lamb	AFL	Troch	AFR
W	L	L		
	W		L	
		W		L

This is the SKB, the 'Skeletal Basis', of the grammar. See Brasoveanu & Prince 2005/11.
Like the other representations of the grammar, it denotes *all* the hierarchies on nGX.CON that select the optima of this language. They are the ones that satisfy all of these (elementary) ranking conditions.

A picture

In some cases, a clean picture can be made from the sparse set:



Such a *Hasse Diagram* is available when there is one W per ERC.

In the general case, with ERCs containing multiple W's, there is no Hasse diagram. Graphical representation loses perspicuity, employing either a swirl of arcs indexed by ERC, or multiple diagrams.

And unlike ERC sets, pictures cannot be analyzed by calculation.

- The System
- The Objects of OT
- Optimality
- Analysis

The System

OT analysis takes place within a *system*:

- About a system, we can say things demonstrably true or false.

Optimality is meaningful within a system ⟨S.GEN , S.CON⟩.

An optimal form is better than all distinct candidates. ***All*!**

- No claim of optimality can be validated or refuted without knowledge of the entire candidate set.

● S.GEN must be specified.

The ERC polls the judgments of all constraints. ***All*!**

-No claim of betterness can be validated or refuted without knowledge of the entire constraint set.

● S.CON must be specified.

The Objects of OT

Given an OT system S = ⟨S.GEN ,S.CON⟩,

we obtain the three fundamental objects.

1. Language

2. Grammar

3. Typology

The Objects of OT

1. **Language**

The *candidates* from each candidate set that are **optimal**
under some given hierarchy, a total order on S.CON.

Extensional: a set of linguistic structures and mappings.

2. **Grammar**

The *set* of all hierarchies that yield the same language.

Intensional: a description of the extensional language.

A grammar may be given by a set of **ERCs**
that exactly delimits the set of hierarchies yielding the lg.

Intensional: a description of the extensional language.

The Objects of OT

3. **Typology** of S

Extensional. The *languages* of S.

Intensional. The *grammars* of S.

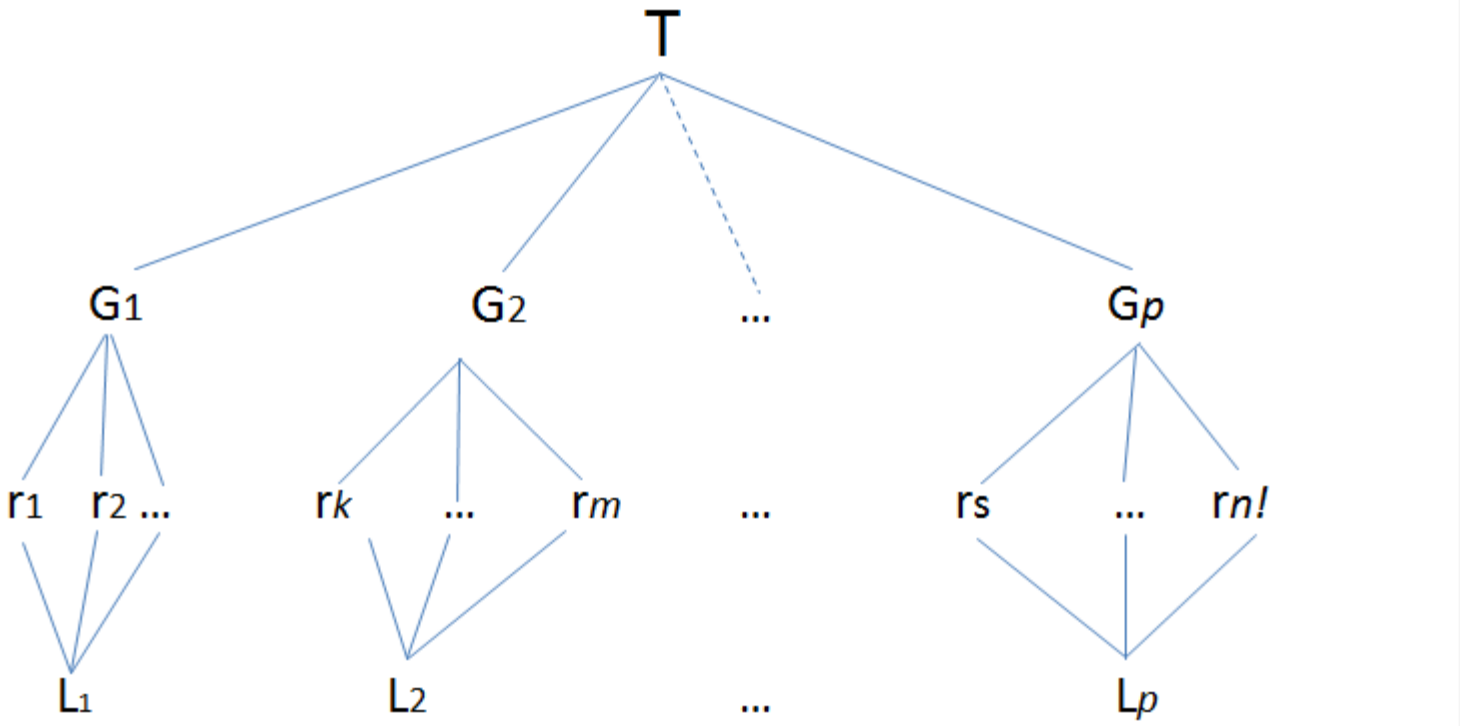
To understand S, we must understand the typology of S.

To understand the typology of S, we must understand its languages and their grammars.

Given ⟨S.GEN, S.CON⟩, the languages & grammars are fixed.

- We must figure out what they are: *analysis*.

The Objects of OT



A typology consists of grammars;
a grammar, of all hierarchies that deliver the same language.

OT analysis: Getting Grammars

1. *OT analysis* therefore aims to arrive at **the grammars** that the system S provides for the data under consideration.

2. We seek the grammar(s) delivering *optima* that accord with the data.

3. We want the grammar(s) in the form that grammars are defined: as a set of ERCs.

4. We are not primarily or directly engaged in 'ranking' the constraints.

5. We are 'arguing optimality'.
   - The ERCs we obtain from optima delimit the set of rankings that constitutes the grammar.

OT analysis: from S to T

1. **After such knowledge**, what forgiveness?
   Once you have fixed S, you have no further choices.

2. **One among many**. The analysis of even one language requires knowledge of the typology of S.

3. **The typology of S** contains the information about which languages and grammars admitted by the system S comport with (more-or-less comport with) the data under analysis.

4. **The analytical situation**. You have the data on one side and the assumptions of S on the other.

Data, meet Theory

1. To justify the claim that the data has *exactly this analysis* under S, you must show that *it has no others* under S.

2. We do not *observe* feet, features, tones, phrases, phases, nodes, labels, links, operators, …, the substance of linguistic representation.

We posit them in S.

How many arrangements yield the same observables?

We may like one, but what does the theory say?

3. Only when we know the *typology* do we obtain a complete, guess-less view of how S characterizes the data.

OT analysis: Getting Typologies

1. Having specified the system S = ⟨S.GEN, S.CON⟩ , we have brought its typology into logical existence. But we do not know what it is.

2. To obtain it, we must assemble a collection of candidate sets (csets), in accordance with S.GEN , sufficient to distinguish all the grammars of T.

3. This sample of csets, if sufficient, is termed a *universal support* for T.

4. A *support* for a grammar is a collection of csets sufficient to yield the entire grammar.

5. A *universal support* is so termed because, when all choices of possible optima are made, one from each cset, we get all the grammars of T.

See the Appendix and Alber, DelBusso, & Prince (2015) for the universal supports of nGX.

OT analysis: Getting Typologies

1. **About Optima**. The individual csets must contain every candidate that is *possibly optimal* --- optimal under some ranking.

- The csets must be *optimum-complete*.
- Because to be optimal is to beat all distinct competitors.

2. **The Big Easy**. Finding a universal support, and establishing its universality, need not be trivial.

- But it is essential.

  Without it, we are open to speculation and self-delusion.
- With it, we can begin to see where the theory leads.

See Prince 2017a for checklist of items appearing in a valid analysis.

What is OT?

The System. S = ⟨S.GEN, S.CON⟩.

The Objects. Language, grammar, typology.

Optimality. Better on a constraint,
Better on a hierarchy of constraints,
Best.

Analysis. S ⇒T ⇒G, an analysis if it exists under S.

Some Downloadable Resources

ERCs, Ranking, OT:

RCD – The Movie. Includes notes on OT fundamentals.

Brasoveanu & Prince. Includes introductory description of ERC logic.

Prince 2002. Develops and explores ERC logic in detail.

Software based on the concepts discussed here:

OTWorkplace. Excel-based calculation of the objects of OT.

nGX – analysis & universal supports:

Alber & Prince, The Book of nGX.

Alber, DelBusso, & Prince. From Intensional Properties to Universal Support.

Prince, OT Checklist

See References for links and more.

References

Alber, B. and A. Prince. 2015, in prep. *Typologies*. Ms. U. Verona and Rutgers U.
Alber, B. and A. Prince. 2015-16. Outline of Property Theory. Ms. U. Verona & Rutgers U.
Alber, B. and A. Prince. 2017. The Book of nGX. ROA-1312.
Alber, B., N. DelBusso, A. Prince. 2015 & To appear. From Intensional Properties to Universal Support. *Language: Phonological Analysis*. ROA-1235.
Brasoveanu, A. and A. Prince. 2005/11. Ranking and Necessity. *NLLT* 29:3-70. ROA-794.
Gigerenzer, G. and G. Goldstein. 1996. Reasoning the fast and frugal way. Psych Rev. 103.4, 650-69.
Hyde, B. 2012. Alignment Constraints. *NLLT* 30: 789-836.
Merchant, N. and A. Prince. 2016. *The Mother of All Tableaux*. ROA-1285.
Prince, A., N. Merchant, and B. Tesar. 2017. OTWorkplace. Updates & info.
Prince, A. 2002. *Entailed Ranking Arguments*. ROA-500.
Prince, A. 2009. RCD – The Movie. ROA-1057
Prince, A. 2013. Metrical Theory as a Portal on Theory. YouTube.
Prince, A. 2017a. OT Checklist. ROA-1306.
Prince, A. 2017b. Representing OT Grammars. ROA-1309.
Prince, A., N. Merchant, and B. Tesar. 2007-17. OTWorkplace.
Prince, A. and P. Smolensky. 1993/2004. *Optimality Theory*. Blackwell. ROA-537.
Samek-Lodovici, V. and A. Prince. 1999. Optima. ROA-363.
Susskind, L. The theoretical minimum.

Appendix: Universal Supports for nGX

Alber, DelBusso, and Prince (2015) prove that the minimal universal supports for nGX are exactly the following:

- The 3s cset + a cset of any even length greater than 2s.
- Any cset with candidates of an odd length greater than 3s.

'Minimal' means that you can't take away a cset and still have a universal support.

Minimal universal supports for nGX with the shortest candidates:

- The 3 and 4 syllable csets
- The 5 syllable cset

These are shown on the following slides. Only possible optima are included.

A Universal Support for nGX

nGX	output	opt	P-s	lamb	Troch	AFL	AFR
3s	-o-uX-		1	0	1	1	0
	-uX-o-		1	0	1	0	1
	-uX-X-		0	1	2	2	1
	-o-Xu-		1	1	0	1	0
	-Xu-o-		1	1	0	0	1
	-Xu-X-		0	2	1	2	1
	-X-uX-		0	1	2	1	2
	-X-Xu-		0	2	1	1	2
4s	-o-o-uX-		2	0	1	2	0
	-o-o-Xu-		2	1	0	2	0
	-uX-o-o-		2	0	1	0	2
	-uX-uX-		0	0	2	2	2
	-Xu-o-o-		2	1	0	0	2
	-Xu-Xu-		0	2	0	2	2

A Universal Support for nGX

nGX	output	opt	P-s	lamb	Troch	AFL	AFR
5s	-o-o-o-uX-		3	0	1	3	0
	-o-o-o-Xu-		3	1	0	3	0
	-o-uX-uX-		1	0	2	4	2
	-uX-o-o-o-		3	0	1	0	3
	-uX-uX-o-		1	0	2	2	4
	-uX-uX-X-		0	1	3	6	4
	-o-Xu-Xu-		1	2	0	4	2
	-Xu-o-o-o-		3	1	0	0	3
	-Xu-Xu-o-		1	2	0	2	4
	-Xu-Xu-X-		0	3	1	6	4
	-X-uX-uX-		0	1	3	4	6
	-X-Xu-Xu-		0	3	1	4	6

Du mußt dein Leben ändern.

Mural by Jonathan Horowitz, Highland Park, NJ. jonathanjacobhorowitz.com