Comparing positional licensing patterns in HG and OT*

Eric Baković and Anna Mai
University of California, San Diego

1 Introduction

Positional licensing refers to the observation that elements (e.g. particular feature values or feature value combinations) can be limited to specific positions (e.g. syllable onsets, initial syllables, stressed syllables, etc.). These elements are said to be *licensed in* those positions. A simplified schematic example is shown in Figure 1, with just one licensing position $\mathcal{P}$ among the set of all positions $\mathcal{E}$. The element x is licensed in $\mathcal{P}$, but not elsewhere in $\mathcal{E}$; we will conventionally refer to the complement of $\mathcal{P}$ as $\hat{\mathcal{P}}$.

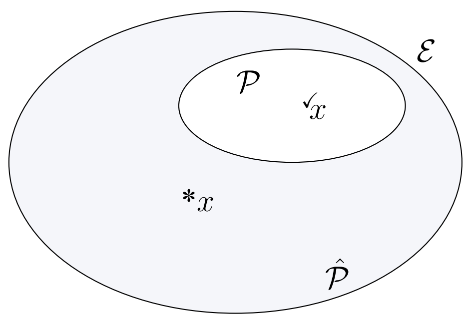

Figure 1: Positional licensing
$\mathcal{E}$ = all positions; $\mathcal{P}$ = licensing position(s); $\hat{\mathcal{P}}$ = non-licensing position(s); x = $\mathcal{P}$-licensed element(s)

Positional licensing patterns have been analyzed using either positional markedness or positional faithfulness constraints (Beckman, 1997; Lombardi, 2001; Zoll, 2004). The effect of a positional markedness constraint is schematically represented on the left in Figure 2: the constraint $*x/\hat{\mathcal{P}}$ is violated when the element x occurs in $\hat{\mathcal{P}}$ — that is, outside of the licensing position $\mathcal{P}$. The effect of a positional faithfulness constraint is schematically represented on the right: the constraint $*x \mapsto \emptyset/\mathcal{P}$ is violated when x is mapped unfaithfully in the licensing position $\mathcal{P}$. (Unfaithful mappings are represented by deletion throughout.)

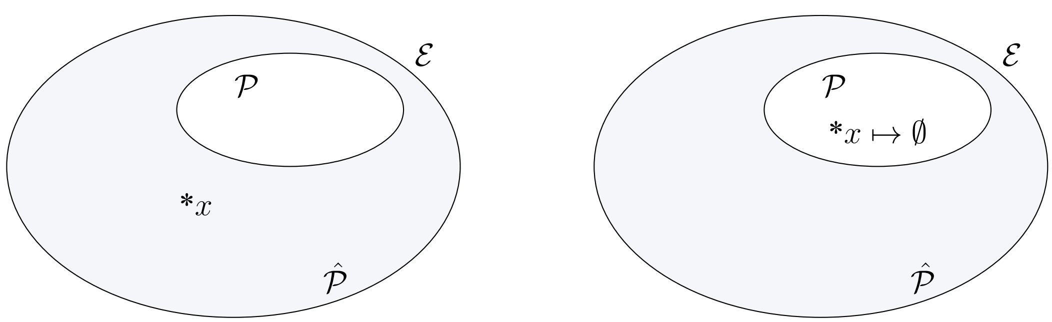

Figure 2: Positional markedness (left) and positional faithfulness (right)

Jesney (2016) investigates the typologies of different positional licensing *systems* in OT (Prince &

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Smolensky, 2004) and HG (Legendre et al., 1990).[1] She finds that when there are two (or more) licensing positions, $\mathcal{P}_1$ and $\mathcal{P}_2$, two additional types of licensing pattern are possible depending on (a) whether licensing is analyzed with markedness or faithfulness and (b) on the form of constraint interaction, OT ranking or HG weighting. One of these additional licensing pattern types is *disjunctive*: x is licensed when it is *either* in $\mathcal{P}_1$ *or* in $\mathcal{P}_2$ (or in both; disjunction is inclusive). The other type of licensing pattern is *conjunctive*: x is licensed only when it is *both* in $\mathcal{P}_1$ *and* in $\mathcal{P}_2$. These two patterns are schematically depicted in Figure 3.

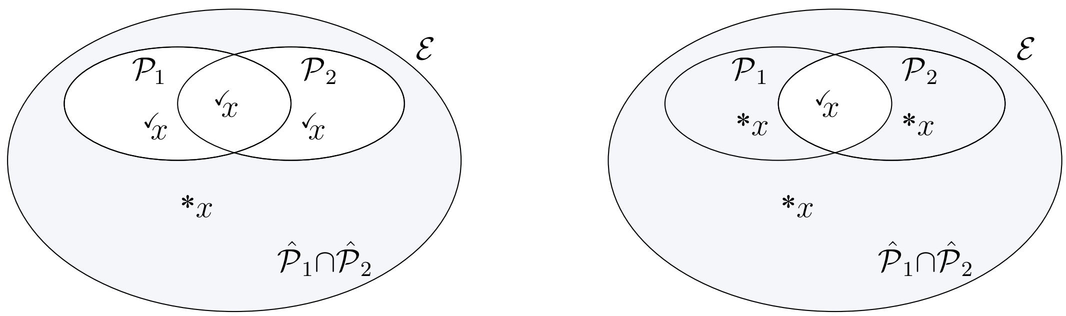

Figure 3: Disjunctive licensing (left) and conjunctive licensing (right)

All of Jesney's systems consist of candidate sets with two structures each, one faithfully realizing the input element x and the other unfaithfully deleting x. A general markedness constraint $*x$ is violated by the faithful structure and a general faithfulness constraint $*x \mapsto \emptyset$ is violated by the unfaithful structure in every case. (Given the lack of ambiguity, we henceforth abbreviate $*x \mapsto \emptyset$ to $*\emptyset$.) The systems differ in privileging up to three fully intersectable licensing positions.[2] Additional candidate sets correspond to the various possible additional fates for x: faithfully realized or deleted in each licensing position or intersection of licensing positions. The systems also differ in whether every licensing position $\mathcal{P}_n$ is associated with a positional markedness constraint $*x/\hat{\mathcal{P}}_n$, a positional faithfulness constraint $*\emptyset/\mathcal{P}_n$, or both.

For the OT systems, Jesney (2016) finds that positional markedness predicts conjunctive licensing patterns while positional faithfulness predicts disjunctive licensing patterns; with both types of constraints, both conjunctive and disjunctive licensing patterns are predicted. For the HG systems, on the other hand, Jesney (2016) finds that both conjunctive and disjunctive licensing patterns are predicted regardless of whether there are only one or both of positional markedness and positional faithfulness constraints in the system. The claims resulting from Jesney's (2016) investigation are summarized in Figure 4.

system	**licensing with OT ranking**		**licensing with HG weighting**	
positional markedness	conjunctive	$\mathbb{M}^{\mathrm{OT}}$	conjunctive & disjunctive	$\mathbb{M}^{\mathrm{HG}}$
positional faithfulness	disjunctive	$\mathbb{F}^{\mathrm{OT}}$	conjunctive & disjunctive	$\mathbb{F}^{\mathrm{HG}}$
positional markedness & positional faithfulness	conjunctive & disjunctive	$\mathbb{S}^{\mathrm{OT}}$	conjunctive & disjunctive	$\mathbb{S}^{\mathrm{HG}}$

Figure 4: Summary of claims from Jesney (2016)

$\mathbb{M}$, $\mathbb{F}$, $\mathbb{S}$ refer to systems with only positional markedness constraints, only positional faithfulness constraints, or both, respectively; superscript 'OT' or 'HG' refers to the mode of constraint interaction (ranking or weighting, respectively)

To explore how Jesney's (2016) generalizations scale with larger numbers of licensing positions and the concomitant increase in possible conjunctive and disjunctive environments, we created sets of positional

1 An OT or HG system "is specified by defining its constraints and the structures they evaluate" (Alber et al., 2016: e88).

2 Consider the positions in the example of English /h/ licensing (Jesney, 2016: §6.2.3): /h/ may be in an initial syllable, a stressed syllable, or a syllable onset; it may be in any intersection of two of these positions (initial stressed syllable, initial syllable onset, or stressed syllable onset); or it may be in the intersection of all three (initial stressed syllable onset).

licensing systems with 2, 3, and 4 licensing positions, as described in Section 2. Despite the suggestion in Figure 4 that the set of patterns predicted by the $\mathbb{S}^{\mathrm{OT}}$ system is equivalent to the set of patterns predicted by the $\mathbb{M}/\mathbb{F}/\mathbb{S}^{\mathrm{HG}}$ systems, we demonstrate in Section 3 that these predictions diverge in deep but structured ways once there are more than two licensing positions. In Section 4, we propose an account for this structured divergence based on 3-position systems, and confirm the validity of that account with an analysis of 4-position systems. In Section 5, we describe how conjoined constraints impact positional licensing patterns, and in doing so provide a counter-example to a claim made in our previous work (Mai & Baković, 2020): whereas the typological predictions of an HG system that includes conjoined constraints and the typological predictions of the corresponding OT system with conjoined constraints are equivalent, predictive equivalence cannot be guaranteed between an HG system *without* conjoined constraint and the corresponding OT system *with* conjoined constraints. In Section 6 we conclude.

2 Systems

We began by analyzing 10 systems, differing along two dimensions: the number (0–3) of licensing positions $\mathcal{A}, \mathcal{B}, \mathcal{C}$ referenced by constraints, and whether those are markedness ($\mathbb{M}$), faithfulness ($\mathbb{F}$), or both ($\mathbb{S}$). Again, the three positions $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are fully intersectable; e.g. if $\mathcal{A}$ = 'initial syllable', $\mathcal{B}$ = 'syllable onset', and $\mathcal{C}$ = 'stressed syllable', then $\mathcal{A} \cap \mathcal{B} \cap \mathcal{C}$ = 'initial stressed syllable onset' (recall footnote 2). The constraint sets for these 10 systems are summarized in Figure 5.

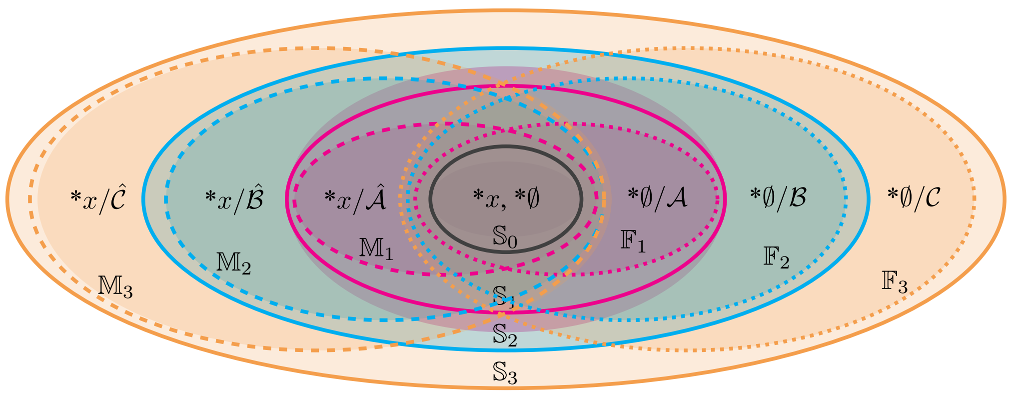

Figure 5: Constraint sets of systems with 0–3 licensing positions

All candidate sets contain two structures, one with input x realized faithfully and the other with x deleted unfaithfully. Having no positional constraints, system $\mathbb{S}_0$ effectively has only one candidate set. Systems $\mathbb{M}_1$, $\mathbb{F}_1$, and $\mathbb{S}_1$ have two candidate sets each: $x \in \mathcal{A}$ and $x \in \hat{\mathcal{A}}$. Systems $\mathbb{M}_2$, $\mathbb{F}_2$, and $\mathbb{S}_2$ have four candidate sets each: $x \in \mathcal{A}$, $x \in \mathcal{B}$, $x \in \mathcal{A} \cap \mathcal{B}$, and $x \in \hat{\mathcal{A}} \cap \hat{\mathcal{B}}$. Finally, systems $\mathbb{M}_3$, $\mathbb{F}_3$, and $\mathbb{S}_3$ each have eight candidate sets: $x \in \mathcal{A}$, $x \in \mathcal{B}$, $x \in \mathcal{C}$, $x \in \mathcal{A} \cap \mathcal{B}$, $x \in \mathcal{A} \cap \mathcal{C}$, $x \in \mathcal{B} \cap \mathcal{C}$, $x \in \mathcal{A} \cap \mathcal{B} \cap \mathcal{C}$, and $x \in \hat{\mathcal{A}} \cap \hat{\mathcal{B}} \cap \hat{\mathcal{C}}$.

3 Patterns

Factorial typologies under OT and HG were computed and analyzed using OTWorkplace (Prince et al., 2007–2020). HG was simulated with the equalizer algorithm of Mai & Baković (2020) and confirmed with OT-Help (Staubs et al., 2010); but see the Appendix for a qualification. Nine sets of patterns are predicted, summarized graphically in Figure 6 (for 0–2 licensing positions) and Figure 7 (for 3 licensing positions).

❶ $\left\{\mathbb{S}_0^{\mathrm{OT}}, \mathbb{S}_0^{\mathrm{HG}}\right\} = x \in \mathcal{E}$

❸ $\mathbb{M}_2^{\mathrm{OT}} = \left\{ \text{❷} +,\; x \in \mathcal{B},\; x \in \mathcal{A} \cap \mathcal{B} \right\}$ (last member in dashed box)

❷ $\left\{ \mathbb{M}/\mathbb{F}/\mathbb{S}_1^{\mathrm{OT}},\; \mathbb{M}/\mathbb{F}/\mathbb{S}_1^{\mathrm{HG}} \right\} = \left\{ \text{❶} +,\; x \in \mathcal{A} \right\}$

❹ $\mathbb{F}_2^{\mathrm{OT}} = \left\{ \text{❷} +,\; x \in \mathcal{B},\; \boxed{x \in \mathcal{A} \cup \mathcal{B}} \right\}$

❺ $\left\{ \mathbb{S}_2^{\mathrm{OT}},\; \mathbb{M}/\mathbb{F}/\mathbb{S}_2^{\mathrm{HG}} \right\} =$ ❸ $\cup$ ❹

[dashed box] *conjunctive licensing*

[solid box] *disjunctive licensing*

Figure 6: Summary of patterns with 0–2 licensing positions

With 0–2 licensing positions (Figure 6 above), OT and HG predict the same typologies. The 0-position systems $\mathbb{S}_0^{\text{OT}}$ and $\mathbb{S}_0^{\text{HG}}$ both unsurprisingly predict just two patterns: one in which x is licensed (= faithfully realized) everywhere and another in which it is not licensed (= unfaithfully deleted) anywhere. All of the 1-position systems, $\mathbb{M}/\mathbb{F}/\mathbb{S}_1^{\text{OT}}$ and $\mathbb{M}/\mathbb{F}/\mathbb{S}_1^{\text{HG}}$, predict those same two patterns as well as a pattern in which x is licensed only in the single licensing position $\mathcal{A}$. These three patterns are in turn predicted by all of the 2-position systems, as well as a pattern in which x is licensed only in the second licensing position $\mathcal{B}$. But this is where the predictions of the different systems begin to diverge in the manner found by Jesney (2016); recall Figure 4. $\mathbb{M}_2^{\text{OT}}$ predicts a conjunctive licensing pattern, where x is licensed only in the intersection of the two licensing positions, $\mathcal{A}\cap\mathcal{B}$. $\mathbb{F}_2^{\text{OT}}$ instead predicts a disjunctive licensing pattern, where x is licensed in the union of the two licensing positions, $\mathcal{A}\cup\mathcal{B}$. Lastly, both $\mathbb{S}_2^{\text{OT}}$ and all of the 2-position HG systems ($\mathbb{M}/\mathbb{F}/\mathbb{S}_2^{\text{HG}}$) predict both the conjunctive (intersection) and the disjunctive (union) licensing patterns.

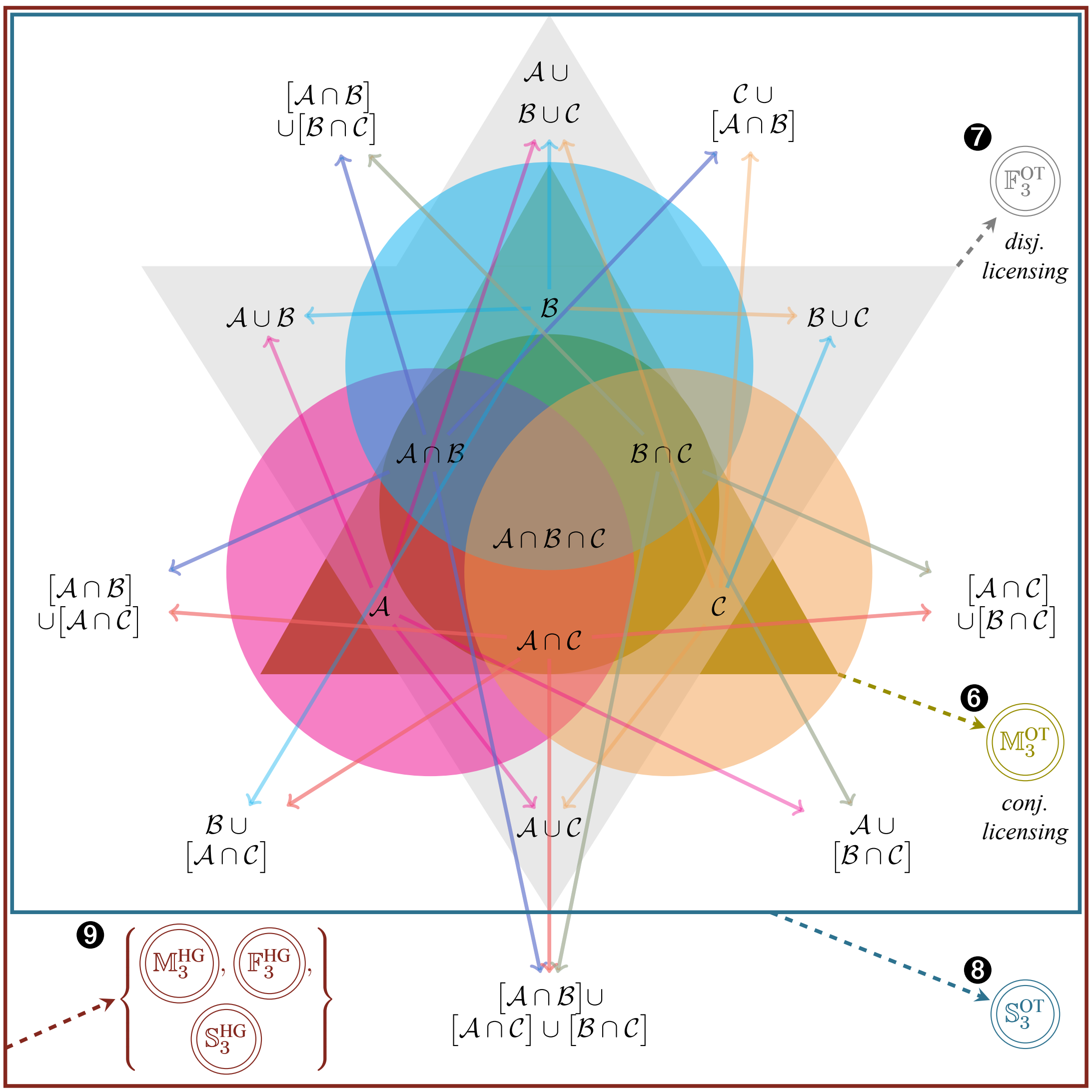

Figure 7: Summary of patterns with 3 licensing positions

With 3 licensing positions (Figure 7 above), a deeper split between the OT and HG systems becomes apparent. To start, all 3-position systems predict licensing everywhere, licensing nowhere, and licensing in any one of the three positions $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$. The positional markedness OT system $\mathbb{M}_3^{\text{OT}}$ in addition predicts all of the conjunctive licensing patterns with positional intersection ($\mathcal{A}\cap\mathcal{B}$, $\mathcal{A}\cap\mathcal{C}$, $\mathcal{B}\cap\mathcal{C}$, $\mathcal{A}\cap\mathcal{B}\cap\mathcal{C}$), while the positional faithfulness OT system $\mathbb{F}_3^{\text{OT}}$ predicts all of the disjunctive licensing patterns with positional union

($\mathcal{A}\cup\mathcal{B}$, $\mathcal{A}\cup\mathcal{C}$, $\mathcal{B}\cup\mathcal{C}$, $\mathcal{A}\cup\mathcal{B}\cup\mathcal{C}$). The full OT system $\mathbb{S}_3^{\text{OT}}$ adds to all of these patterns a set of disjunctive licensing patterns where one or both of the members of the union is an intersection ($\mathcal{A}\cup[\mathcal{B}\cap\mathcal{C}]$, $\mathcal{B}\cup[\mathcal{A}\cap\mathcal{C}]$, $\mathcal{C}\cup[\mathcal{A}\cap\mathcal{B}]$, $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{B}\cap\mathcal{C}]$, $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$, $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]$), and all of the 3-position HG systems ($\mathbb{M}/\mathbb{F}/\mathbb{S}_3^{\text{HG}}$) predict all of these patterns plus the 3-way union-of-intersections pattern $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$.

4 Analysis

Jesney (2016:194) also observes that the 3-way union-of-intersections pattern noted at the end of the previous section is a prediction unique to the 3-position HG systems, the one pattern in twenty that is not predicted by the most inclusive 3-position OT system $\mathbb{S}_3^{\text{OT}}$, and concludes from this 5% discrepancy that "[t]ypological expansion due to cumulative constraint interaction is quite limited" Jesney (2016: 195).

Our goal in this section is to explain why this pattern is missing from $\mathbb{S}_3^{\text{OT}}$, which otherwise predicts 2-way union-of-intersections patterns (§4.1), and how it is that all of the 3-position HG systems predict it (§4.2). We confirm our explanation by widening our view to consider 4-position systems (§4.3), and conclude that the typological expansion due to cumulative constraint interaction is better described as *structured*, not limited.

4.1 *2-way union-of-intersection patterns in* $\mathbb{S}_3^{\text{OT}}$ An arbitrary member of the set of three 2-way union-of-intersection patterns, $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{B}\cap\mathcal{C}]$, is schematically illustrated in Figure 8, along with the OT constraint ranking responsible for it. The first thing to note is that, given that there are only three positions and two intersections in the pattern, there is guaranteed to be a unique position that is common to both intersections (in this case, $\mathcal{B}$). We call this unique shared position the *anchor*, and the remaining positions the *complement*. Moving from the top of the constraint ranking down, we can see that the presence of a unique anchor is critical to predicting a union-of-intersections pattern in $\mathbb{S}_3^{\text{OT}}$ — and thus why the 3-way union-of-intersections pattern $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$ is not predicted, given that it does not have a unique anchor.

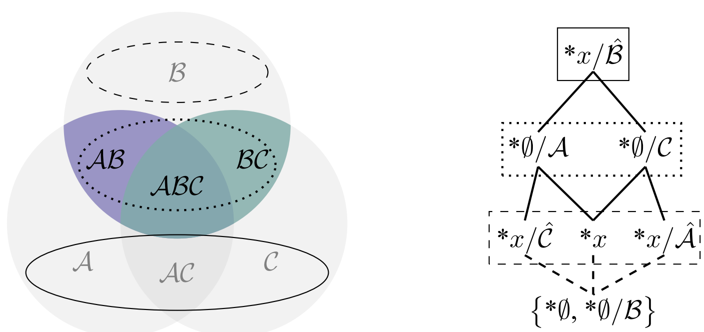

Figure 8: Schematic illustration (left) and ranking (right) for $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{B}\cap\mathcal{C}]$

Starting from the top of the ranking on the right in Figure 8: the positional markedness constraint that references the anchor, $*x/\hat{\mathcal{B}}$, excludes x from the subset of the complement $\mathcal{A}$ and $\mathcal{C}$ that does not intersect with the anchor. This relationship between the ranking and the illustration is indicated by the solid lines around both in Figure 8. The positional faithfulness constraints that reference the complement, $*\emptyset/\mathcal{A}$ and $*\emptyset/\mathcal{C}$, then serve to protect x in the intersections between the complement and the anchor, $\mathcal{A}\cap\mathcal{B}$ (abbreviated $\mathcal{AB}$ in the illustration) and $\mathcal{B}\cap\mathcal{C}$ (abbreviated $\mathcal{BC}$). This relationship is indicated by dotted lines in Figure 8. Finally, the remaining markedness constraints $*x/\hat{\mathcal{A}}$, $*x/\hat{\mathcal{C}}$, and $*x$ exclude x from the non-intersecting subset of the anchor, $\mathcal{B}$. This relationship is indicated by dashed lines in Figure 8.

The other two 2-way union-of-intersections patterns, $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]$ and $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$, are the same in relevant respects and are analyzed with the same ranking, *mutatis mutandis*. This is again because in each case there is a unique anchor position, allowing for the cascade of positional filtering described for Figure 8 above: exclude x from the subset of the complement that does not intersect with the anchor, protect x in the intersections between the complement and the anchor, and exclude x from the subset of the anchor that does not intersect with the complement. Without a unique anchor position, as in the 3-way union-of-intersections pattern $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$, the cascade fails to gain footing on the entirety of the complement. This becomes clear when we consider how the 3-positions HG systems are all able to predict this pattern.

4.2 *3-way union-of-intersections pattern in* $\mathbb{M}/\mathbb{F}/\mathbb{S}_3^{\mathrm{HG}}$ The illustration of the 3-way union-of-intersections pattern in Figure 9 makes it clear that there is no anchor common to the three intersections; each excludes a different one of the three positions. Cumulative interaction of positional markedness constraints (in the $\mathbb{M}/\mathbb{S}_3^{\mathrm{HG}}$ systems) or of positional faithfulness (in the $\mathbb{F}/\mathbb{S}_3^{\mathrm{HG}}$ systems) is needed to describe this pattern.

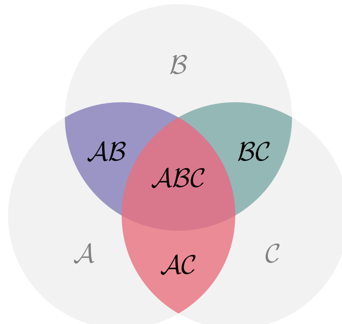

Figure 9: Schematic illustration for $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$

There are many and varied constraint weighting conditions in the $\mathbb{S}_3^{\mathrm{HG}}$ system that will describe the pattern in Figure 9, so we focus here on the more unified conditions in the $\mathbb{M}_3^{\mathrm{HG}}$ and $\mathbb{F}_3^{\mathrm{HG}}$ systems.[3] First, $\mathbb{M}_3^{\mathrm{HG}}$. Recall that in this system there is only one, general faithfulness constraint (*$\emptyset$) and the full array of markedness constraints, general (*x) and positional (*$x/\hat{\mathcal{A}}$, *$x/\hat{\mathcal{B}}$, and *$x/\hat{\mathcal{C}}$). The weight of the faithfulness constraint must be greater than the weight of each of the markedness constraints in order to allow x to surface at all (1a), but the cumulative weights of each possible pairing of positional markedness constraints must in turn be greater than the weight of the faithfulness constraint in order to penalize instances of x surfacing in any single position that does not intersect with at least one other position (1b).

(1) Weighting conditions for $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$ in $\mathbb{M}_3^{\mathrm{HG}}$

a. $w(*\emptyset) > \{w(*x), w(*x/\hat{\mathcal{A}}), w(*x/\hat{\mathcal{B}}), w(*x/\hat{\mathcal{C}})\}$

b. $\{[w(*x/\hat{\mathcal{A}})+w(*x/\hat{\mathcal{B}})], [w(*x/\hat{\mathcal{A}})+w(*x/\hat{\mathcal{C}})], [w(*x/\hat{\mathcal{B}})+w(*x/\hat{\mathcal{C}})]\} > w(*\emptyset)$

Note that the cumulative weight of each pair of positional markedness constraints in (1b) is crucial. Each of the positional markedness constraints on its own is too specific to penalize x when it surfaces in any non-intersecting position; for example, *$x/\hat{\mathcal{A}}$ on its own can only penalize x when it surfaces anywhere outside of licensing position $\mathcal{A}$. The general markedness constraint, on the other hand, is *too* general, penalizing x everywhere. Acting together, though, two positional markedness constraints give just the right result. *$x/\hat{\mathcal{A}}$ and *$x/\hat{\mathcal{B}}$ penalize x when it surfaces anywhere outside the union of the licensing positions $\mathcal{A}$ and $\mathcal{B}$ — that is, in licensing position $\mathcal{C}$ or elsewhere in the set of all positions $\mathcal{E}$. The other two pairings of positional markedness constraints complete the picture, leaving only the positional intersections as possible licensors.

Recall now that in the $\mathbb{F}_3^{\mathrm{HG}}$ system there is only one, general markedness constraint (*x) and the full array of faithfulness constraints, general (*$\emptyset$) and positional (*$\emptyset/\mathcal{A}$, *$\emptyset/\mathcal{B}$, and *$\emptyset/\mathcal{C}$). The weight of the markedness constraint must be greater than the weight of each of the faithfulness constraints in order to prevent x from surfacing in any single position (2a), but the cumulative weights of each possible pairing of positional faithfulness constraints must in turn be greater than the weight of the markedness constraint in order to allow instances of x to surface in any of the three positional intersections (2b).

(2) Weighting conditions for $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$ in $\mathbb{F}_3^{\mathrm{HG}}$

a. $w(*x) > \{w(*\emptyset), w(*\emptyset/\mathcal{A}), w(*\emptyset/\mathcal{B}), w(*\emptyset/\mathcal{C})\}$

b. $\{[w(*\emptyset/\mathcal{A})+w(*\emptyset/\mathcal{B})], [w(*\emptyset/\mathcal{A})+w(*\emptyset/\mathcal{C})], [w(*\emptyset/\mathcal{B})+w(*\emptyset/\mathcal{C})]\} > w(*x)$

The cumulative interactions in (2b) are again crucial, but for a slightly different reason in this case even though they look very similar to the previous case in (1b). Each of the faithfulness constraints on their own,

[3] Recall that the constraint set of $\mathbb{S}_3^{\mathrm{HG}}$ is the union of the constraint sets of $\mathbb{M}_3^{\mathrm{HG}}$ and $\mathbb{F}_3^{\mathrm{HG}}$, so any patterns describable by the latter two systems is at least describable by the former, more inclusive system with a similar set of weighting conditions.

both general and positional, are too general to allow x to surface only in positional intersections; for example, $*\emptyset/\mathcal{A}$ on its own protects x when it surfaces anywhere in the licensing position $\mathcal{A}$. But acting together, two positional faithfulness constraints do the trick: $*\emptyset/\mathcal{A}$ and $*\emptyset/\mathcal{B}$ protect x from deletion anywhere in the intersection of the licensing positions $\mathcal{A}$ and $\mathcal{B}$. The other two pairings of positional faithfulness constraints complete the picture, again rendering only the positional intersections as possible licensors.

4.3 *4-position systems* The unique anchor position analysis discussed in §4.1 is confirmed by further analysis of 4-position systems. A unique anchor position is shared in each of the union-of-intersections patterns predicted by $\mathbb{S}_4^{\mathrm{OT}}$ (46 of 96 total patterns predicted), and no unique anchor position is shared in any of the additional union-of-intersections patterns predicted by $\mathbb{M}/\mathbb{F}/\mathbb{S}_4^{\mathrm{HG}}$ (54 of 150 total patterns predicted). One example of each type of pattern is shown in Figure 10: on the left is the $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{A}\cap\mathcal{D}]$ pattern, anchored in licensing position $\mathcal{A}$, and on the right is the unanchored $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$ pattern.

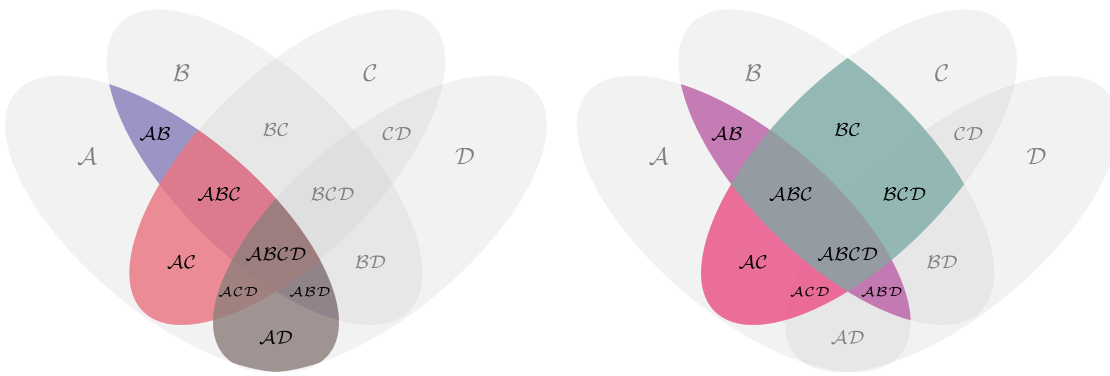

Figure 10: Schematic illustrations for anchored (left) and unanchored (right) 3-way union-of-intersections patterns in 4-position systems

The ranking of constraints in $\mathbb{S}_4^{\mathrm{OT}}$ required for the anchored $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{A}\cap\mathcal{D}]$ pattern is as expected: first $*x/\hat{\mathcal{A}}$ excludes x from the subset of the complement $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ that does not intersect with the anchor, then the positional faithfulness constraints that reference the complement $*\emptyset/\mathcal{B}$, $*\emptyset/\mathcal{C}$, and $*\emptyset/\mathcal{D}$ serve to protect x in the intersections between the complement and the anchor, and finally the remaining markedness constraints $*x$, $*x/\hat{\mathcal{B}}$, $*x/\hat{\mathcal{C}}$, and $*x/\hat{\mathcal{D}}$ exclude x from the non-intersecting subset of the anchor.

The unanchored $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]$ pattern requires cumulative constraint interaction. As before, there are many and varied constraint weighting conditions in the $\mathbb{S}_4^{\mathrm{HG}}$ system that will describe this pattern, so we focus here on the more unified conditions in the $\mathbb{M}_4^{\mathrm{HG}}$ and $\mathbb{F}_4^{\mathrm{HG}}$ systems (recall footnote 3). In $\mathbb{M}_4^{\mathrm{HG}}$, the cumulative weights of the positional markedness constraint pairs $*x/\hat{\mathcal{A}}$ & $*x/\hat{\mathcal{B}}$, $*x/\hat{\mathcal{A}}$ & $*x/\hat{\mathcal{C}}$, and $*x/\hat{\mathcal{B}}$ & $*x/\hat{\mathcal{C}}$ must be greater than the weight of the general faithfulness constraint $*\emptyset$, which must in turn be greater than the weight of each of the individual markedness constraints and the cumulative weights of the other positional markedness constraint pairs ($*x/\hat{\mathcal{A}}$ & $*x/\hat{\mathcal{D}}$, $*x/\hat{\mathcal{B}}$ & $*x/\hat{\mathcal{D}}$, $*x/\hat{\mathcal{C}}$ & $*x/\hat{\mathcal{D}}$). In $\mathbb{F}_4^{\mathrm{HG}}$, the cumulative weights of the positional faithfulness constraint pairs $*\emptyset/\mathcal{A}$ & $*\emptyset/\mathcal{B}$, $*\emptyset/\mathcal{A}$ & $*\emptyset/\mathcal{C}$, and $*\emptyset/\mathcal{B}$ & $*\emptyset/\mathcal{C}$ must be greater than the weight of the general markedness constraint $*x$, which must in turn be greater than the weight of each of the individual faithfulness constraints and the cumulative weights of the other positional faithfulness constraint pairs ($*\emptyset/\mathcal{A}$ & $*\emptyset/\mathcal{D}$, $*\emptyset/\mathcal{B}$ & $*\emptyset/\mathcal{D}$, $*\emptyset/\mathcal{C}$ & $*\emptyset/\mathcal{D}$).

As already noted above, less than two-thirds of the patterns predicted by the 4-position HG systems are predicted by the most inclusive 4-position OT system, $\mathbb{S}_4^{\mathrm{OT}}$ (96 of 150). Broadening our scope even wider to 5-position systems, about one in five patterns predicted by the HG systems are predicted by $\mathbb{S}_5^{\mathrm{OT}}$ (669 of 3287). It's clearly not the case that the typological consequences of cumulative constraint interaction are limited, at least not numerically or proportionally — instead they are *structurally delimited*, with uniquely shared anchor positions in union-of-intersections patterns forming the basis of that delimiting structure.

5 Cumulative interaction and conjoined constraints

The previous two sections identified how the typological predictions of $\mathbb{S}_n^{\text{OT}}$ diverge from those of $\mathbb{M}/\mathbb{F}/\mathbb{S}_n^{\text{HG}}$ due to the differential ability of HG and OT to capture combinations of conjunctive and disjunctive licensing environments. However, as shown in Mai & Baković (2020), when conjoined constraints[4] are introduced into these frameworks, the differences in their typological predictions disappear. We tested this result by augmenting each of the systems described in Section 2 such that all possible conjunctions of the positional markedness constraints (in the $\mathbb{M}_n$ systems), all possible conjunctions of the positional faithfulness constraints (in the $\mathbb{F}_n$ systems), or both (in the $\mathbb{S}_n$ systems) are added to the constraint set.[5] So, for example, the $\mathbb{F}_3^{\text{OT}}$ and $\mathbb{F}_3^{\text{HG}}$ systems contain the constraints $\{*x, *\emptyset, *\emptyset/\mathcal{A}, *\emptyset/\mathcal{B}, *\emptyset/\mathcal{C}\}$; we augmented this set to $+\mathbb{F}_3$ such that it also contains the conjunctions $\{*\emptyset/\mathcal{AB}, *\emptyset/\mathcal{AC}, *\emptyset/\mathcal{BC}, *\emptyset/\mathcal{ABC}\}$.

Contrary to Mai & Baković's (2020) claim that the addition of conjoined constraints to an HG grammar has no effect on the typology it predicts, we found that conjoined constraints can expand both HG and OT typologies under particular conditions, and that the additional patterns in the expanded typologies elucidate the ways that conjoined constraints behave in constraint-based systems. In particular, the $\mathbb{M}/\mathbb{F}/\mathbb{S}_4^{\text{HG}}$ systems all predict the same 150 patterns, and the corresponding OT systems without conjoined constraints produce subsets of those patterns: $\mathbb{M}/\mathbb{F}_4^{\text{OT}}$ both predict the same 17 patterns, and $\mathbb{S}_4^{\text{OT}}$ predicts those same 17 plus 79 more (= 96). Augmenting these systems with conjoined constraints in the manner described above, all six augmented systems predict the same set of 168 patterns, 18 in addition to the 150 predicted by $\mathbb{M}/\mathbb{F}/\mathbb{S}_4^{\text{HG}}$.

The 18 patterns generated by the conjoined constraint systems that are absent from $\mathbb{M}/\mathbb{F}/\mathbb{S}_4^{\text{HG}}$ (and by extension also absent from $\mathbb{M}/\mathbb{F}/\mathbb{S}_4^{\text{OT}}$) are those for which the weighting conditions of the base set of non-conjoined constraints are contradictory or otherwise infeasible. For example, one such pattern in $+\mathbb{F}_4$ licenses x in the following union of intersections: $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{D}]$. To realize x in any of the three conjunctive licensing environments requires the cumulative weight of each pair of positional faithfulness constraints corresponding to the intersecting positions to be greater than the weight of the general markedness constraint $*x$. These conditions are shown in (3a–c). To prevent x from occurring in other positional intersections, the weight of $*x$ must be greater than that of the general faithfulness constraint $*\emptyset$ and greater than that of each of the remaining pairs of positional faithfulness constraints. These conditions are shown in (3d–f).

(3) Subset of weighting conditions necessary for $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{D}]$ in $\mathbb{F}_4^{\text{HG}}$

a. $w(*\emptyset/\mathcal{A})+w(*\emptyset/\mathcal{C}) > w(*x)$

b. $w(*\emptyset/\mathcal{B})+w(*\emptyset/\mathcal{C}) > w(*x)$

c. $w(*\emptyset/\mathcal{B})+w(*\emptyset/\mathcal{D}) > w(*x)$

d. $w(*x) > \{w(*\emptyset), w(*\emptyset/\mathcal{A})+w(*\emptyset/\mathcal{B})\}$

e. $w(*x) > \{w(*\emptyset), w(*\emptyset/\mathcal{A})+w(*\emptyset/\mathcal{D})\}$

f. $w(*x) > \{w(*\emptyset), w(*\emptyset/\mathcal{C})+w(*\emptyset/\mathcal{D})\}$

The problem becomes apparent on closer inspection of these weighting conditions. Consider conditions (3a,b,d): together, they state that the weight of $*\emptyset/\mathcal{C}$ can be added to the weight of either $*\emptyset/\mathcal{A}$ (3a) or $*\emptyset/\mathcal{B}$ (3b) to overtake $*x$, but that the combined weight of $*\emptyset/\mathcal{A}$ and $*\emptyset/\mathcal{B}$ is insufficient to the task (3d). This means that the weight of $*\emptyset/\mathcal{C}$ must be greater than the individual weights of $*\emptyset/\mathcal{A}$ and $*\emptyset/\mathcal{B}$, leading to a contradiction between conditions (3c) and (3f): if the combined weight of $*\emptyset/\mathcal{B}$ and $*\emptyset/\mathcal{D}$ is sufficient to overtake $*x$ (3c), and the weight of $*\emptyset/\mathcal{C}$ must be greater than the weight of $*\emptyset/\mathcal{B}$ as we surmised from (3a,b,d), then the combined weight of $*\emptyset/\mathcal{C}$ and $*\emptyset/\mathcal{D}$ must be greater than the weight of $*x$, contrary to (3f). Conditions (3b,c,f) together lead to the same contradiction between conditions (3a) and (3d). These contradictions are succinctly captured by algebraic rearrangement of the inequalities in (3), as shown in (4):

[4] The violation domain for a conjoined constraint is the entire candidate structure. Repeating footnote 4 from Mai & Baković (2020: 4): we refrain from calling these conjoined constraints *local conjunctions*, as the 'local' delimiter is meant to invoke the (relative) locality of conjunct violations in domains smaller than the full candidate structure. See Legendre et al. (2006) and Pater (2016) for discussion of the non-equivalence of HG and OT with local conjunction.

[5] The restriction to conjunctions of only positional constraints, and to only conjunctions of constraints within the same family (markedness or faithfulness), is in part due to how conjoined constraints work and in part due to the highly restricted nature of the candidate sets in these systems. First, any candidate that violates a positional constraint also violates the corresponding general constraint, so a conjunction of these is not distinct from the positional constraint itself. Second, every candidate set in our systems only has two candidates one of which violates some subset of the markedness constraints and the other of which violates some subset of the faithfulness constraints, thus leaving no opportunity for cumulative/conjoined interactions between markedness and faithfulness.

(4) Contradictions in the weighting conditions for $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{D}]$ in $\mathbb{F}_4^{\text{HG}}$

a. From (3a) and (3d): $w(*\emptyset/\mathcal{C}) > w(*x) - w(*\emptyset/\mathcal{A}) > w(*\emptyset/\mathcal{B})$

b. From (3c) and (3f): $w(*\emptyset/\mathcal{B}) > w(*x) - w(*\emptyset/\mathcal{D}) > w(*\emptyset/\mathcal{C})$

c. From (3c) and (3d): $w(*\emptyset/\mathcal{D}) > w(*x) - w(*\emptyset/\mathcal{B}) > w(*\emptyset/\mathcal{A})$

d. From (3a) and (3f): $w(*\emptyset/\mathcal{A}) > w(*x) - w(*\emptyset/\mathcal{C}) > w(*\emptyset/\mathcal{D})$

The weighting conditions in (4a) and (4b) contradict one another, requiring both that $w(*\emptyset/\mathcal{C}) > w(*\emptyset/\mathcal{B})$ and that $w(*\emptyset/\mathcal{B}) > w(*\emptyset/\mathcal{C})$. Similarly, the conditions in (4c) and (4d) contradictorily require $w(*\emptyset/\mathcal{D}) > w(*\emptyset/\mathcal{A})$ and $w(*\emptyset/\mathcal{A}) > w(*\emptyset/\mathcal{D})$. For this reason, the pattern licensing x in the union-of-intersections $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{D}]$ cannot be generated with this base set of constraints.

Conjoined constraints offer an escape hatch for these apparent conflicts, however, changing the weighting conditions given in (3a) and (3d) into those given in (5).

(5) Subset of weighting conditions for $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{D}]$ in $+\mathbb{F}_4^{\text{HG}}$

a. $w(*\emptyset/\mathcal{AC}) + w(*\emptyset/\mathcal{A}) + w(*\emptyset/\mathcal{C}) > w(*x)$

b. $w(*\emptyset/\mathcal{AB}) + w(*\emptyset/\mathcal{A}) + w(*\emptyset/\mathcal{B}) > w(*x)$

The conjoined constraints $*\emptyset/\mathcal{AC}$ and $*\emptyset/\mathcal{AB}$ prevent the conditions in (5) from requiring that $w(*\emptyset/\mathcal{C}) > w(*\emptyset/\mathcal{B})$ as shown in (4a). Instead, $w(*\emptyset/\mathcal{C})$ must be greater than $w(*x) - w(*\emptyset/\mathcal{AC}) - w(*\emptyset/\mathcal{A})$, and $w(*\emptyset/\mathcal{B})$ must be less than $w(*x) - w(*\emptyset/\mathcal{AB}) - w(*\emptyset/\mathcal{A})$. Similarly, the presence of conjoined constraints $*\emptyset/\mathcal{BD}$ and $*\emptyset/\mathcal{CD}$ in the system avert the conclusion of (4b) that $w(*\emptyset/\mathcal{B}) > w(*\emptyset/\mathcal{C})$. In this way, the addition of conjoined constraints to an HG system circumvents infeasible weighting conditions to enable the expression of patterns that would otherwise not be generable.

Similarly, the $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{D}]$ union-of-intersections pattern is not generable in OT without conjoined constraints because the conjunctive licensing environments require conflicting rankings of the base constraints. For instance, permitting the expression of x in the environment $[\mathcal{A}\cap\mathcal{C}]$ requires either $*\emptyset$, $*\emptyset/\mathcal{A}$, or $*\emptyset/\mathcal{C}$ to dominate $*x$, but prohibiting x in environments $\mathcal{A}$ and $\mathcal{C}$ individually requires that $*x$ dominate $*\emptyset$, $*\emptyset/\mathcal{A}$, and $*\emptyset/\mathcal{C}$. It is for this reason that even the well-anchored union-of-intersections pattern $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{A}\cap\mathcal{D}]$ is not among the 17 patterns generated by $\mathbb{M}/\mathbb{F}_4^{\text{OT}}$: neither positional markedness nor positional faithfulness constraints alone are capable of modelling the expression of x in an intersection of positions without also allowing the expression of x in each of the component positions individually.

However, when both positional markedness and positional faithfulness constraints are included in a system, as they are in $\mathbb{S}_4^{\text{OT}}$, the anchored union-of-intersections pattern $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{A}\cap\mathcal{D}]$ is generable, as discussed in Section 4. The inclusion of an additional set of positional constraints in $\mathbb{S}_4^{\text{OT}}$ results in a greater number of disjunctive ranking conditions (ERCs with multiple Ws), providing the necessary latitude to achieve intersections without entailing each of their component positions individually. Returning to our example with $[\mathcal{A}\cap\mathcal{C}]$, in a system with positional faithfulness constraints alone, prohibiting the expression of x in environments $\mathcal{A}$ and $\mathcal{C}$ individually requires $*x$ be ranked over $*\emptyset$, $*\emptyset/\mathcal{A}$, and $*\emptyset/\mathcal{C}$. Only six rankings of these four constraints are possible, and all of them conflict with the ranking conditions that make the expression of x in $[\mathcal{A}\cap\mathcal{C}]$ possible. With both positional markedness and positional faithfulness constraints, the number of possible rankings that prevent x from being realized uniquely in either $\mathcal{A}$ or $\mathcal{C}$ is much greater. Preventing the realization of x in $\mathcal{A}$ can be achieved by ranking $*x$, $*x/\mathcal{B}$, $*x/\mathcal{C}$, or $*x/\mathcal{D}$ over $*\emptyset$ and $*\emptyset/\mathcal{A}$, and preventing the realization of x in $\mathcal{C}$ can be achieved by ranking $*x$, $*x/\mathcal{A}$, $*x/\mathcal{B}$, or $*x/\mathcal{D}$ over $*\emptyset$ and $*\emptyset/\mathcal{C}$. Because of these additional ranking options, the subranking required by $\mathbb{F}_4^{\text{OT}}$ ($*x \gg \{*\emptyset, *\emptyset/\mathcal{A}, *\emptyset/\mathcal{C}\}$) does not occur in rankings that generate $[\mathcal{A}\cap\mathcal{B}]\cup[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{A}\cap\mathcal{D}]$ in $\mathbb{S}_4^{\text{OT}}$; the subranking $*\emptyset/\mathcal{C} \gg \{*x, *\emptyset, *\emptyset/\mathcal{A}\}$ occurs instead. In this way, the inclusion of both positional markedness and positional faithfulness constraints increases the combinatorial possibilities that lead to particular input-output pairs.

Returning to the unanchored union-of-intersections pattern $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{D}]$: despite the increased flexibility of the $\mathbb{S}_4^{\text{OT}}$ system, without an anchor, no positional faithfulness constraint can be ranked over general markedness without undermining at least one of the intersecting licensing environments. This conflict lies in the content of the positional constraints rather than their combinatorics. In $+\mathbb{F}_4^{\text{OT}}$, conjoined constraints solve this conflict by targeting particular positional intersections explicitly; they have precisely the right content to license certain conjunctive environments without undermining others. And indeed, it is the specificity of their content, rather than their interaction with other constraints in the system that allows conjoined constraints to capture unachored union-of-intersection patterns. Unlike what we see with the interaction of the full set of constraints in $\mathbb{S}_4^{\mathrm{OT}}$, the interaction of conjoined constraints with the base set is not greater than the sum of its parts: the subrankings that made $[\mathcal{A}\cap\mathcal{C}]$ impossible in $\mathbb{F}_4^{\mathrm{OT}}$ still feature in the rankings responsible for $[\mathcal{A}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{C}]\cup[\mathcal{B}\cap\mathcal{D}]$ in $+\mathbb{F}_4^{\mathrm{OT}}$. The only difference is that now a conjoined constraint can rank where the other half of a contradiction would have. Conjoined constraints thus mediate conditions that would otherwise require impossible rankings or weightings.

6 Conclusion

We have demonstrated two things in this paper. The first builds on the insights of Jesney (2016), who noted the typological differences between OT and HG systems with positional constraints, as summarized in Figure 4. Jesney's focus was on how either positional markedness or positional faithfulness constraints are sufficient to generate patterns that require both families of positional constraints in OT. We've shown here that there are deep and structured typological differences between these HG systems and the corresponding, most inclusive OT system ($\mathbb{S}^{\mathrm{OT}}$), differences that are revealed once there are positional licensing constraints in those systems that refer to more than two licensing positions. We've also shown that there is a basis for those differences: if a given union-of-intersections pattern involves a common position across the intersections — an anchor — there is a way to generate that pattern in OT; with no anchor, the added power of cumulative constraint interaction in HG is required.

The second thing that we have shown concerns the claim made in Mai & Baković (2020) that conjoined constraints engender no expansion in the typological expressivity of HG. While it remains the case that the addition of conjoined constraints to HG and OT systems with a common base set of constraints "equalizes" their predictive differences, conjoined constraints do so in a way that is distinct from the impact that cumulative constraint interaction has on HG typologies. Adding conjoined constraints to HG and OT systems expands their expressive capacities by introducing the means to sidestep conflicting weighting and ranking conditions. Our future work will investigate the formal consequences of this expansion in greater depth.

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