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Sympathy Theory and the Set of Possible Winners

Abstract

In a recent paper Samek-Lodovici and Prince (1999) show: (i) that all the potential winners (harmonically unbounded candidates) can be determined in a ranking independent way, and (ii) that this set of potential winners is finite in number. However, they did not consider the influence of sympathetic constraints (McCarthy, 1999, 2003) on their results. These constraints can promote perpetual losers to the set of potential winners. With sympathetic constraints, the finiteness of the set of potential winners is therefore in question. Also, the violations assigned by sympathetic constraints are indirectly ranking dependent (via the choice of the sympathetic candidate). This paper shows that: (i) the set of potential winners is still finite even in a version of OT with sympathetic constraints, and (ii) that the harmonically bounded candidates that sympathetic constraints can promote to the set of potential winners, can be determined in a ranking independent way. It follows that Samek-Lodovici and Prince's results are also valid in an OT grammar with sympathy constraints.

This is an important result for two reasons: (i) If for any given input the set of potential winners were to be infinite, then an infinite typology would be predicted—there will be infinitely many possible languages. However, if the set of potential winners is finite, then only a finite typology is predicted—i.e. it results in a much more restrictive theory. (ii) If the set of potential winners can only be determined in a ranking dependent manner, then the grammar of every language (a ranking of CON) has to consider the full infinite candidate set. However, if the finite set of potential winners can be determined without recourse to a specific grammar (a specific ranking of CON), then it is in principle possible to weed out the perpetual losers before the grammar of a specific language comes into play. The grammar of any given language then needs to consider only the finite set of potential winners.

1. Introduction: Samek-Lodovici and Prince (1999) and Sympathy Theory

One of the central tenets of OT is that the set of candidates that compete for optimal status is infinite in number. This is a particularly appealing feature of OT. Together with a few other theoretical primitives (such as richness of the base and freedom of analysis) the infinity of the candidate set safeguards the theory against opting for an easy way out. It forces the theory to express universals and language particular generalizations by way of constraint interaction rather than by brute force stipulation. If a certain candidate or output pattern is never attested, this must follow from constraint interaction—the same device that is responsible for deciding between possible winners. However, at the same time the concept of an infinite number of candidates that must be evaluated every time an input-output mapping is considered, is also problematic.

Samek-Lodovici and Prince (1999) suggest a very appealing solution to this conundrum. They formulate the conditions for a candidate to be bounded—to be non-optimal under any ranking. The two most important aspects of their results, are that they show (i) that the set of perpetual losers can be identified in a ranking independent way (i.e. it is not grammar/language specific), and (ii) that the remainder of the candidate set (the set of potential winners) is finite in number. The set of candidates that needs to be considered for optimal status can therefore universally be reduced considerably before EVAL comes into play.

Samek-Lodovici and Prince present their argument within a simplified version of classical OT. In classical OT all constraints have strictly local vision, where "local" means that a constraint sees only the candidate it evaluates (and the input if it is a faithfulness constraint) when it decides whether to assign violations to the candidate. A constraint has no global vision, in the sense that it is blind to all other constraints in the system, as well as to all other candidates. This is one of the key reasons for the success of Samek-Lodovici and Prince's arguments. The violations a constraint assigns can be determined without recourse to the rest of the hierarchy—i.e. without reference to ranking.

However, in recent years several extensions to OT have been proposed in which constraint evaluation is not so straightforwardly ranking independent anymore. Among these extensions are Sympathy Theory (McCarthy, 1999, 2003), Output-Output correspondence (Benua, 2000), preference constraints (Coetzee, 2000), and targeted constraints (Bakovic and Wilson, 2000). In each of these extensions to classical OT the vision of at least some constraints has been broadened so that they can see candidates besides the candidate under evaluation, and/or other constraints, and/or the ranking between constraints. The violations assigned by these more global-vision constraints therefore depend more on the ranking between constraints, than what was the case with constraints in classical OT. This paper investigates the implications of this move away from strictly local-vision constraints for the results of Samek-Lodovici and Prince. In particular the influence of Sympathy Theory on their results will be considered. Sympathy Theory was chosen from this group for the following reasons: (i) It is one of the more fully worked out of these non-local theories. (ii) Sympathetic constraint interaction (or something like it) seems to be truly required to account for certain opaque phenomena in language. The existence of sympathy constraints can therefore not be ignored. (iii) Sympathy Theory was formulated specifically with the view to promote perpetual losers to potential winners. It can therefore be expected that Sympathy Theory will interact in interesting ways with Samek-Lodovici and Prince's results.

In the version of Sympathy Theory used in this paper there are two sympathetic constraints, both of which have global vision in the sense described above. These constraints are ❀CUMUL and ❀DIFF. The violations these constraints assign depend on the candidate that is chosen as the sympathetic candidate. They assign violations based on a comparison between the faithfulness violations of the sympathetic candidate and the candidate under consideration. These constraints should therefore be able to "see" other candidates (the sympathetic candidate), and also to see other constraints (specifically the faithfulness constraints), and to compare the violations of different candidates in terms of faithfulness constraints. In addition to this, the choice of the sympathetic candidate is based on the ranking of the constraints in the specific grammar, and since the violations of the sympathetic constraints are assigned in relation to the sympathetic candidate, their evaluation seems to be ranking dependent even if in an indirect way. The result of the violations assigned by ❀CUMUL and ❀DIFF is that certain candidates that were perpetual losers can be promoted to the set of potential winners.

From this discussion it is clear that Samek-Lodovici and Prince's results cannot straightforwardly be translated into a version of OT that allows for sympathetic constraint interaction. Both of their key results are in danger. Since the violations assigned by the sympathy constraints are indirectly (via the sympathetic candidate) ranking dependent, it is not clear whether the set of potential winners can be determined in a ranking independent way anymore. Also, since ❀CUMUL and ❀DIFF can promote a candidate from the set of perpetual losers to the set of potential winners, the set of potential winners is not necessarily finite anymore. There are two questions that need to be answered with regard to Samek-Lodovici and Prince's results in an OT grammar that allows sympathy constraints: (i) Can the set of potential winners still be determined in a ranking independent way? (ii) Is the set of potential winners still finite? In this paper it is argued that both of these questions should be answered affirmatively.

It is argued in this paper that it is indeed possible to determine the set of candidates that ❀CUMUL can promote from being perpetual losers to being potential winners in a ranking independent way.[1] This is done in three incremental steps, by showing that: (i) The set of potential sympathetic candidates can be determined in a ranking independent way. (ii) The violations assigned by ❀CUMUL relative to each possible sympathetic candidate can be determined in a ranking independent way. (iii) The conditions that must be met for ❀CUMUL to promote a perpetual loser to the set of potential winners can be stated in a ranking independent fashion.

It is also argued that the set of potential winners is still finite, even taking into consideration the candidates that ❀CUMUL promotes to this set.

The paper is presented in the following sections: In §2 an explanation of the most important details of Samek-Lodovici and Prince's results is offered. The next section (§3) contains an explanation of the specific version of Sympathy Theory that is assumed in this paper. In this section it also shown that Sympathy Theory does indeed have consequences for Samek-Lodovici and Prince's results. Section §4 comprises the central part of this paper. In this section the requirements that must be met for the sympathy constraint ❀CUMUL to promote a perpetual loser to the set of potential winners are developed step-by-step. Throughout this section focus is on the fact that these requirements make no reference to ranking. Section §5 uses the results of §4 to show that the set of potential winners is still finite even after the ❀CUMUL-promoted candidates have been added to this set. Finally, in §6 the implications of this paper are summarised and evaluated. A few suggestions are also offered for how to proceed further.

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1 There is an intricate relation between the violations assigned by ❀CUMUL and those assigned by ❀DIFF. These two constraints can jointly or separately promote a candidate from the set of perpetual losers to the set of potential winners. Only the ways in which ❀CUMUL on its own can do this, are investigated in this paper. The contribution that ❀DIFF can make is left for future research. (For more on this see §3.2 and the conclusion in §6.)

2. Samek-Lodovici and Prince on perpetual losers and bounding sets

This section contains a discussion of the relevant results of Samek-Lodovici and Prince's paper, and is not a repetition of the details and technicalities of how they reach these results. For that the reader is referred is to their paper.[2]

Three aspects of Samek-Lodovici and Prince's paper are of crucial importance in this paper. These aspects are: (i) that the set of harmonically bounded candidates can be determined in a ranking independent manner; (ii) that a harmonically bounded candidate can be bounded by more than one bounding set; (iii) that the set of non-harmonically bounded candidates (i.e. the possible winners) is finite in number. These three aspects are discussed shortly below.

Identifying the set of harmonically bounded candidates. Samek-Lodovici and Prince show that there are some candidates emitted by GEN that can never be optimal under any ranking of the constraints in CON, and that these candidates can be identified without recourse to the ranking between the constraints in CON. In an OT grammar, EVAL compares candidates according to a language specific ranking of CON to find the optimal candidate. However, if certain candidates can never be optimal under any ranking of CON, and if there is a way to weed these candidates out before EVAL comes into play (i.e. in a language independent way), then EVAL does not have to consider these perpetual loser candidates.

A perpetual loser has this status because under any ranking there will always be at least one other candidate that is better than the perpetual loser—that bounds the loser under that ranking. It is possible for a perpetual loser to have different candidates as bounders under different rankings. The candidates that together bound a perpetual loser then form its **bounding set**. Membership of a bounding set can be determined in a ranking independent manner. Samek-Lodovici and Prince (1999:9) give the following definition of a bounding set:

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[2] A detailed knowledge of Samek-Lodovici and Prince's paper is not required to follow the general gist of the present paper. All that is required is that the reader be willing to accept the results of Samek-Lodovici and Prince's paper.

(1) **Bounding set**

Let K be the infinite set of candidates emitted by GEN, and $z \in K$, and Σ all the constraints in CON. Then $B(z) \subset K$[3] is a bounding set for candidate z, iff:

- **Strictness**. Every member of B is better than z on at least one constraint in Σ.
- **Reciprocity**. If z is better than some member of B on a certain constraint $C \in \Sigma$, then some other member of B beats z on C.

Every candidate that has a non-empty bounding set is a perpetual loser and does not have to be considered by EVAL. Since nothing in the definition of a bounding set refers to ranking between constraints, it means that the set of candidates can be scaled down to only the set containing possible winners in a language/grammar independent way. The grammar of each language will then in actual fact have to consider for every input-output mapping only those candidates that are actual possible winners.

One candidate can have more than one bounding set. Since it is possible that a bounded candidate can have more than one bounder per possible ranking, it is possible that a single bounded candidate can have more than one bounding set. This is illustrated with an example below.

(2) **Candidate z has more than one bounding set**

	M_1	M_2	F_1	F_2
z	*	*	*	*
a		**	*	
b		*	*	
c	**			*

3 Samek-Lodovici and Prince (1999: 9) requires that the B(z) simply be a subset and not a proper subset of K—i.e. they use "$B(z) \subseteq K$" instead of "$B(z) \subset K$". This is impossible. Since $z \in K$, it follows that the only way in which the set B(z) can be equal to the set K, is when $z \in B(z)$. And this is never possible—a candidate can never be in its own bounding set! This would require through strictness that z be better than itself on at least one constraint. Here and in the rest of this paper, it is therefore assumed that a bounding set is a proper subset of the full candidate set.

In this example, candidate z is a perpetual loser, a harmonically bounded candidate. It is bounded by the maximal bounding set B(z) = {a, b, c}. It can be checked that this set satisfies the requirements for a bounding set. Strictness requires that each of the members of B(z) be better than z on at least one constraint. Both a and b beat z on M_1 and F_2. Candidate c beats z on M_2 and F_1. Strictness is therefore satisfied for all three members of B(z). Now consider reciprocity. Candidate z beats c on M_1, but is in turn beaten by a and b on this constraint. Similarly z beats a on M_2, but is beaten by c. For candidates a and c reciprocity is therefore satisfied. Candidate z does not beat b on any constraint. For candidate b reciprocity is vacuously satisfied. The set {a, b, c} qualifies as a bounding set for candidate z. It is correctly predicted that there is no ranking of the constraints in this tableau in terms of which candidate z can be optimal.

However, closer inspection of this tableau will show that certain subsets of B(z), namely {b}, {a, c}, {a, b} and {b, c}, also satisfy the requirements for a bounding of z. This shows that a single bounded candidate can be bounded by several bounding sets. This issue will become relevant later in this paper when the conditions under which a bounded candidate can be liberated from its bounding set will be investigated. To liberate a bounded candidate, it will be necessary to liberate it from all of its bounding sets.

The set of potential winners is finite in number. Samek-Lodovici and Prince's argument about the finitude of the set of possible winners depends on two assumptions about an OT grammar: (i) an OT grammar is a complete ranking of the constraints in CON, and (ii) the idea of grammatically distinct candidates. In OT candidates are defined in terms of their constraint violations. It is (at least theoretically) possible that two formally distinct candidates can have identical constraint violation profiles. An OT grammar will not be able to distinguish between two such candidates. Samek-Lodovici and Prince show that the number of grammatically distinct possible winners is finite.

When the candidates under consideration are limited to only grammatically distinct candidates, then every grammar (every total ranking of the constraints) can select only one candidate as optimal for any given input. Since the number of possible rankings between the constraints is finite, it follows that the number of grammatically distinct possible winners, is finite. For *n* constraints, the number of possible winners can be maximally *n*!, which can be extremely large but never infinite.

So, it is possible to determine the set of possible winners in a ranking independent way, and this set can be shown to be finite in number. It follows that EVAL has to consider only a finite number of candidates every time an input-output mapping is considered.

3. Basic notions of Sympathy Theory

Several versions of Sympathy Theory have been proposed in the literature. All of them use the same basic concepts, but differ in the specific ways they choose to implement these concepts. The most important versions of Sympathy Theory are those proposed by McCarthy (1999, 2003), De Lacy (1998), Ito and Mester (2003) and Walker (2000). In this paper a slightly simplified version of McCarthy's Sympathy Theory is adopted. A basic familiarity with McCarthy's version of Sympathy Theory is assumed in this paper. Therefore, only the most relevant details of this theory are mentioned here, with focus on how it differs from other versions of Sympathy Theory and on the simplifying assumptions made in this paper. After these basic comments, it is shown that Sympathy Theory is relevant for harmonic bounding. One example is discussed of where ❀CUMUL promotes a bounded candidate to the set of potential winners, and one example of where ❀DIFF does this.

3.1 Sympathy Theory: the basics

Faithfulness constraints are the only possible selectors. This differs from other versions of Sympathy Theory in which markedness constraints (De Lacy, 1998; Ito and Mester, 2003) or a stratified constraint hierarchy (Walker, 2000) can select the sympathetic candidate.

Sympathy is mediated through cumulative faithfulness. The correspondence between the candidate under evaluation and the sympathetic candidate is evaluated indirectly via the constraints ❀CUMUL and ❀DIFF, instead of with ordinary correspondence (faithfulness) constraints. McCarthy (1999) uses both possibilities, but rejects sympathetic correspondence for cumulativity. These two constraints are defined as follows (see McCarthy, 1999: 353):

(3) **Sympathy constraints**

Let ❀*cand* be the sympathetic candidate and *cand* the candidate under evaluation.

a. ❀CUMUL

Cand must accumulate the faithfulness violations of ❀*cand*. This means that the faithfulness violations of *cand* should be a superset of the faithfulness violations of ❀*cand*. (This constraint is categorical.)

b. ❀DIFF

Every faithfulness violation incurred by *cand* is also incurred by ❀*cand*. (This means that *cand* earns a violation for every one of its faithfulness violations not shared by ❀*cand*.)

The idea that sympathy effects are mediated via cumulative faithfulness is not shared by all versions of Sympathy Theory. De Lacy (1998), Ito and Mester (2003),Walker (2000) and Jun (1999) all use direct correspondence relations between the sympathetic candidate and the output to mediate sympathy effects.

Only one selector per grammar. It is assumed that only one faithfulness constraint per grammar can act as selector. This implies that there can also be only one sympathetic candidate, and therefore only one set of sympathetic constraints per grammar. McCarthy (1999) has shown with an example from Yokuts that it is indeed possible and necessary to allow multiple selectors, sympathetic candidates and sympathetic constraints per grammar. In a language with more than one opaque process, multiple sympathetic interactions might be required. This assumption is necessary in this paper in order to simplify the discussion. See in the conclusion (§6) for a discussion of the implications of this simplification for the results of this paper.

Only one pair of ❀CUMUL and ❀DIFF in every grammar. If several selectors are allowed, then several sets of the sympathy constraints will be required in the grammar, each indexed for the selector constraint that it is related to. Since only one selector is allowed in the version of Sympathy Theory assumed in this paper, there is only one pair of the sympathy constraints, and there is no need to keep track of which selector they are related to. It is assumed that these two constraints are present in all grammars, even in grammars in which no sympathetic constraint interaction occurs. If no faithfulness constraint acts as a selector, then no sympathetic candidate will be chosen, and therefore the sympathy constraints will be inactive—they will be vacuously satisfied by all candidates.

This last point turns out to be important for the discussion in the rest of this paper. A grammar with sympathy constraints but with no designated selector constraint is equivalent to a classic OT grammar without sympathy constraints. In a classic OT grammar the violation marks assigned by the constraints are independent from the ranking between constraints. This is not necessarily true of a grammar with sympathy constraints, where the violations of the sympathy constraints depend on the choice of the sympathetic candidate, which in turn depends on ranking. However, if the sympathy constraints are inactive because there is no sympathetic candidate, they will assign zero violation marks to all candidates under all rankings. In grammars without a selector constraint, all constraint evaluation can therefore be done without any need to refer to ranking. Samek-Lodovici and Prince's results therefore hold for this special subgroup of grammars in a Sympathy Theoretic version of OT.

The candidates that are perpetual losers in this special subgroup of grammars, are therefore exactly the candidates that would have been losers in a classic OT grammar. To determine the influence of sympathy constraints on Samek-Lodovici and Prince's results, is therefore equivalent to determining the influence of activating a selector constraint in this special subgroup of sympathy grammars. To determine the set of losers for a sympathy grammar, it is therefore possible to first determine the set of losers in accordance with Samek-Lodovici and Prince's bounding set definition in (1), and then to determine which of these losers will be promoted to winner status by the addition of sympathy constraints.

3.2 That sympathy matters for harmonic bounding

Both ❀CUMUL and ❀DIFF can promote a bounded candidate z to the set of potential winners. In both instances they achieve this by removing from z's maximal bounding set B(z) enough members that the residue does not qualify as a bounding set anymore. This is illustrated below first with an example of ❀CUMUL liberating z from its bounding set, and then an example of ❀DIFF liberating z. The purpose of this discussion is simply to show that sympathy can change the set of potential winners, and that it does therefore have implications for Samek-Lodovici and Prince's results. The examples will therefore not be discussed in detail—that is deferred to later in the paper (§4).

In the example below candidate z is bounded by candidate b, i.e. B(z) = {b}. Candidate ❀c is chosen as sympathetic candidate by selector constraint ✯F_1. Constraint M_1 is added simply to ensure that ❀c is not also a bounder for z.

(4) **❀CUMUL liberating z from its bounding set**[4]

	❀CUMUL	M_1	M_2	✭F_1	F_2	❀DIFF
☞ z			**¡	*	*	*
☜ b	*!		*	*		*
❀ c		*!		✓	*	

The faithfulness violations of bounded candidate z forms a superset of that of sympathetic candidate ❀c. Candidate z therefore satisfies ❀CUMUL. However, the faithfulness violations of candidate b and candidate ❀c are disjoint. Candidate b therefore violates ❀CUMUL. The result of this is that z no longer has a bounding set. There was only one member in B(z), namely candidate b. In terms of the constraint ❀CUMUL, candidate z is better than candidate b and there is no other member of B(z) that can be better than z to satisfy the reciprocity requirement for bounding sets. Candidate b therefore looses its membership of B(z), and z has an empty bounding set. With the ranking in the tableau above, the bounded candidate z is then chosen as optimal. A perpetual looser z has been promoted to be a potential winner, because of the fact that ❀CUMUL has destroyed its bounding set.

❀DIFF can never liberate a candidate that is bounded by a single candidate. The reason: If some candidate z is bounded by a single candidate b, it means that there is no constraint on which the bounder b can have more violations than bounded candidate z. This is because there are no other candidates in the bounding set to satisfy the reciprocity requirement for bounding sets. For candidate b to be worse than bounded candidate z on ❀DIFF, it is necessary that b be worse than z on at least one faithfulness constraint. This is obviously not possible. However, when candidate z is jointly bounded by more than one candidate, then a candidate in z's bounding set can be worse than z on some constraint—as there are other members in B(z) that can beat z on the specific constraint to satisfy the reciprocity requirement. The example below therefore necessarily is more complicated than that used with regard to ❀CUMUL above. In this tableau candidate z is jointly bounded by two candidates, a and b—i.e. B(z) = {a, b}. Candidate ❀c is chosen as sympathetic candidate by selector ✭F_3.

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4 In this and all further tableaux in this paper: ❀ = sympathetic candidate or sympathetic constraint; ☜ = the candidate that would have won had it not been for sympathetic constraints; ¡ = the violation of the bounded candidate that would have been fatal had it not been for the sympathetic constraints; ✭ = the selector constraint; ✓ = constraint satisfaction.

(5) **❀DIFF liberating z from its bounding set**

	M_1	❀CUMUL	❀DIFF	F_1	F_2	✫F_3
☞ z			***	*¡	*	*
a			***	**!		*
☜ b			****!		***	*
❀ c	*!					✓

Candidate a is better than bounded candidate z on constraint F_2, and candidate b beats z on F_1. Both a and b therefore satisfy the strictness requirement for membership of B(z). Candidate z beats a on F_1, but it is in return beaten candidate b on this constraint. Similarly, z beats b on F_2, but is beaten by a on this constraint. Both a and b also satisfy the reciprocity requirement. The set {a, b} therefore qualifies as abounding set for z.

In this tableau the fully faithful candidate is chosen as sympathetic candidate—therefore all candidates satisfy ❀CUMUL. Candidate b has a total of four faithfulness violations and therefore earns four violations of ❀DIFF. Bounded candidate z has only three faithfulness violations and therefore gets only three ❀DIFF-violations. Bounded candidate z is this better than one of the members of its bounding set on this constraint. To satisfy reciprocity it is necessary that another member of B(z) should be better than z on ❀DIFF. However, the other member of B(z), candidate a, earns the same number of ❀DIFF-violations as candidate z. The bounding set of z is therefore broken by ❀DIFF, and there is a ranking under which z is chosen as optimal candidate. Also ❀DIFF can therefore promote a bounded candidate from the set of perpetual losers to the set of possible winners.[5]

4. ❀CUMUL setting free the bounded candidate

How can ❀CUMUL liberate a bounded candidate from the power of its bounding set? The only way that a bounded candidate z can be liberated from its bounding set B(z) is by removing enough members from z's maximal bounding set that the

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5 However, as was explained in footnote 1, in this paper the influence of only ❀CUMUL on boundedness is investigated. All remarks made about sympathy below are therefore only about ❀CUMUL unless it is stated specifically that they also apply to ❀DIFF.

remaining candidates do not qualify as a bounding set anymore. This is also the only way in which ❀CUMUL can liberate a bounded candidate z.

In order to determine whether some bounded candidate z can be liberated from its bounding B(z) by ❀CUMUL, it is necessary to compile, for each potential sympathetic candidate, a separate set all the of all members of B(z) that ❀CUMUL can remove from B(z) with reference to that specific sympathetic candidate. (Remember that the violations assigned by ❀CUMUL depend on the sympathetic candidate.) The compliment in B(z) of each of these sympathetic candidate specific sets must then be taken. If any of these compliment sets in B(z) does not qualify as a bounding set of z anymore, then z has been liberated from the bounds of its bounding set.

Even this is not enough for bounded candidate z to be promoted from a perpetual loser to a potential winner. The choice of the specific sympathetic candidate that is responsible for liberating z from its bounding set depends on ranking. In particular, it depends on the ranking of all the constraints in the system with the exception of the selector constraint and the sympathy constraints. It is possible that the ranking requirements for the specific sympathetic candidate to be chosen as sympathetic candidate are in conflict with the ranking requirements for bounded candidate z to be optimal (after it has been liberated from its bounding set by ❀CUMUL). After it has been shown that the bounding set of bounded candidate z has been broken by ❀CUMUL, it still needs to be shown that there is indeed a ranking in terms of which z is better than all its competitors and the relevant sympathetic candidate is still chosen as sympathetic candidate.

There are therefore two necessary conditions that must be satisfied for ❀CUMUL to promote a bounded candidate to potential winner status. These conditions are:

(6) **Conditions for a bounded candidate z to be promoted to potential winner by ❀CUMUL**

There must be some sympathetic candidate ❀c such that:

a. Relative to ❀c, ❀CUMUL removes enough members from B(z) that the candidates that remain in B(z) do not qualify as a bounding set for z anymore.

b. There is some ranking between the constraints such that ❀c is chosen as sympathetic candidate and such that z beats all of its competitors.

These requirements are discussed in turn below (§4.1 and §4.2 are dedicated to condition (a), and §4.3 to condition (b)). (Appendix A contains a flowchart of the argument followed in this paper, and should be used as an aid when reading the paper.) The aim of this discussion will be to show that it is possible to determine whether each of the conditions is met without reverting to ranking. However, since both of these requirements crucially depend on the sympathetic candidate, it is first necessary to show that it is possible to determine the set of potential sympathetic candidates in a ranking independent way.

The sympathetic candidate is chosen from the set of candidates obeying some faithfulness constraint that acts as the selector. The candidate from this set that is most harmonic in terms of the constraint hierarchy (disregarding the selector constraint and the sympathy constraints) is designated as sympathetic candidate. Every candidate in the set obeying the selector constraint that is not harmonically bounded in terms of the rest of the constraint set, is therefore a potential sympathetic candidate. Samek-Lodovici and Prince have shown that it is possible to determine whether a candidate is harmonically bounded or not in a way that requires no reference to ranking. Therefore it is also possible to determine the set of potential sympathetic candidates in a ranking independent way.

(7) **Determining the set of potential sympathetic candidates**

Let Σ be the set of all constraints excluding the sympathy constraints.

Let $\mathbb{F} \subset \Sigma$ be set of all faithfulness constraints.

A candidate *cand* is a potential sympathetic candidate iff:

> $\exists$ F $\in \mathbb{F}$ such that *cand* does not violate F and *cand* is not harmonically bounded in terms of (Σ - F).

4.1 How ❀CUMUL removes candidates from a bounding set

For the sake of reference, the definition of a bounding set is repeated here:

(8) **Bounding set**

Let K be the infinite set of candidates emitted by GEN, and z $\in$ K, and Σ all the constraints in CON. Then B(z) $\subset$ K is a bounding set for candidate z, iff:

- **Strictness**. Every member of B is better than z on at least one constraint in Σ.
- **Reciprocity**. If z is better than some member of B on a certain constraint C $\in \Sigma$, then some other member of B beats z on C.

How can ❀CUMUL remove some candidate b $\in$ B(z) from z's bounding set? Based on the characteristics of a bounding set, there are two possibilities.

❀CUMUL can cause candidate b not qualify for membership of B(z) either in terms of *strictness* or *reciprocity*. If ❀CUMUL can cause b to not be better than z on at least one constraint,[6] then b does not qualify for membership of B(z) anymore because of the *strictness* clause in the definition of a bounding set. If ❀CUMUL can have the effect that z is better than b on some constraint, without some other member of B(z) beating z on this constraint,[7] then b looses its membership of B(z) on the grounds of *reciprocity*. It turns out that ❀CUMUL can in fact not break the strictness of the bounding set. Consequently, ❀CUMUL's only chance at removing candidate b from B(z) is through reciprocity. In the discussion below it is first shown that ❀CUMUL cannot break the strictness of a bounding set. Then the conditions under which ❀CUMUL can undo the reciprocity of a bounding set are investigated.

Strictness requires that for any candidate b to be an element of B(z), b must be better than z on at least one constraint. It is because strictness only requires that b be better than z on *at least one* constraint, that ❀CUMUL cannot end b's membership of B(z) via a strictness violation. For b to be a member of B(z) it must already be better than z on some constraint prior to the addition of sympathy. Even if it turns out that z is better than b in terms of ❀CUMUL, there will still be at least one constraint in terms of which b is better than z, and b will still satisfy the strictness requirement for membership of B(z).

Now consider reciprocity. McCarthy (1999, 2003) formulates ❀CUMUL as a categorical constraint—see (3) for a formulation of this constraint. A candidate either accumulates the faithfulness violations of the sympathetic candidate or it does not. There is no notion of accumulating to the some degree. ❀CUMUL is even more categorical than other traditional categorical constraints. ONSET also assigns maximally one mark per violation (per onsetless syllable). But since a candidate can contain several syllables, ONSET can assign several marks per candidate. The domain over which ❀CUMUL evaluates, is the entire candidate, and therefore it can assign maximally one violation per candidate. It is precisely this characteristic of ❀CUMUL that enables it to break a bounding set. Samek-Lodovici and Prince treat constraints as functions that impose a partial harmonic ordering on the candidate set.[8] If a constraint can assign more than one violation mark, it can impose a multi-level harmonic ordering on the candidate set.

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6 I.e. if ❀CUMUL can cause the following statement to be true of b and z: $\neg\exists$ constraint C such that $C(b) < C(z)$. (Where C(x) stands for the number of violations assigned by constraint C to candidate x.)

7 I.e. if ❀CUMUL can cause the following statement to be true of b and z: $\exists$ constraint C such that $C(z) < C(b)$ and $\neg\exists$ candidate a $\in$ B(z) such that $C(a) < C(z)$.

8 See also De Lacy (2000) and Coetzee (2000) for similar ideas about constraints.

However, as a constraint that can maximally assign one violation mark, ❀CUMUL can impose only a two level ordering on the candidate set, namely *candidates with no ❀CUMUL-violations ≻ candidates with one ❀CUMUL-violation*.

Suppose that the bounded candidate z accumulates all the faithfulness violations of the sympathetic candidate, and therefore does not violate ❀CUMUL. Suppose further that b ∈ B(z) does violate ❀CUMUL. Bounded candidate z therefore beats b ∈ B(z) on ❀CUMUL. Reciprocity requires that there be some other member of B(z) that does better than z on ❀CUMUL. But ❀CUMUL imposes only a two level harmonic ordering on the candidate set. If z is better than b on ❀CUMUL, then z is on the highest level of the harmonic ordering that ❀CUMUL can impose, and there can be no candidate that does better than z on ❀CUMUL. Whenever a bounded candidate does better than some member of its bounding set on ❀CUMUL, the effect is that this member of the bounding set looses its membership of the bounding set because of the reciprocity clause in the definition of bounding sets.

It is clearly theoretically possible that ❀CUMUL can remove some of the candidates from z's bounding set B(z). But will a situation like that ever be encountered? Will it ever happen that a bounded candidate z does not violate ❀CUMUL while one of its bounders does? The first possible scenario where this seems plausible is where z is chosen as the sympathetic candidate. If this were to happen, then z itself is of course free from the possibility of violation of ❀CUMUL, while at least some of its bounders may violate it. However, z as a bounded candidate can never be chosen as sympathetic candidate. For z to even be considered for this status, there must be some faithfulness constraint that it obeys (a constraint that can then serve as selector). However, if z satisfies some faithfulness constraint, then all of its bounders must also satisfy this same constraint.[9] And then there will be at least one candidate from B(z) that is more harmonic in terms of the rest of the hierarchy. A bounded candidate can therefore never be chosen as sympathetic candidate, and ❀CUMUL can consequently not remove any member from B(z) in this manner.

But there are circumstances in which ❀CUMUL can remove some candidate from B(z). The argument goes as follows: A bounded candidate z has more violations than each of the members of its bounding set B(z) in terms of at

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9 In general, if a bounded candidate z does not violate some constraint C, then this constraint is also not violated by any of the members z's bounding set B(z). To see why, suppose the opposite. Suppose that b, one of z's bounders, does violate C. Then reciprocity requires that there be another bounder of z that is better than z on constraint C. But because z does not violate C, there can be no candidate that is better than z on C.

least one constraint (this follows from the strictness clause in the definition of a bounding set). If at least one constraint in terms of which b ∈ B(z) beats z is some faithfulness constraint F, then ❀CUMUL can end b's membership of B(z). It is then possible that z accumulates the faithfulness violations of the sympathetic candidate, while b does not. The circumstances under which this will happen is stated below:

(9) **Conditions for ❀CUMUL to remove candidates from a bounding set**

Let z be a bounded candidate, B(z) the bounding set of z, b ∈ B(z), and ❀c the sympathetic candidate.

Let $\mathbb{F}$ be the set of faithfulness constraints.

❀CUMUL will end b's membership of B(z) iff:

$\exists$ F $\in \mathbb{F}$ such that F(b) < F(❀c), and for $\forall$ $C_F \in \mathbb{F}$, C_F(❀c) $\leq$ C_F(z)[10].

The requirement that F(b) < F(❀c) assures that the sympathetic candidate ❀c violates the relevant faithfulness constraint F. This in turn assures that F counts for ❀CUMUL. The fact that b does better than sympathetic candidate ❀c on F implies that b violates ❀CUMUL. The second part of the condition requires that bounded candidate z have at least as many violations as sympathetic candidate ❀c in terms of every faithfulness constraint. This assures that bounded candidate z does not violate ❀CUMUL.

It is crucial that no reference is made to ranking in these conditions. It is therefore possible to determine in a ranking independent way whether these conditions are satisfied or not.

As an illustration of this point, consider again tableau (4) from §3.2. It is repeated below without the sympathy constraints. Assume that candidate z is the candidate that has to be optimal. Candidate z is bounded by candidate b.

(10) **Candidate z bounded by candidate b**

(☹ = candidate that should win but does not.)

	M_1	M_2	F_1	F_2
☹ z		**!	*	*
☞ b		*	*	
c	*!			*

10 Where C(x) represent the number of violations that constraint C assigns to candidate x.

Now, to see how ❀CUMUL can end b's membership of z's bounding set B(z), let F_1 act as a selector. The set of candidates obeying this constraint has only one member, i.e. {c}. The sympathetic candidate will be that candidate from this set that is most harmonic in terms of the hierarchy without F_1. Since candidate c is the only member of this set, it follows that it will be the sympathetic candidate. The first requirement for ❀CUMUL to end b's membership of B(z), is that there be some faithfulness constraint in terms of which b beats ❀c. This requirement is met—$F_2(b) = 0 < F_2(❀c) = 1$. The second part of the requirement is that bounded candidate z has at least as many violations as sympathetic candidate ❀c on every faithfulness constraint. Also this requirement is met—$F_1(❀c) = 0 \leq F_1(z) = 1$, and $F_2(❀c) = 1 \leq F_2(z) = 1$. The prediction is therefore that ❀CUMUL should be able to remove candidate b from B(z). This is confirmed by adding the sympathetic constraints to this tableau.

(11) **❀CUMUL liberating z from its bounding set**

	❀CUMUL	M_1	M_2	✯F_1	F_2	❀DIFF
☞ z			**¡	*	*	*
☜ b	*!		*	*		*
❀ c		*!		✓	*	

In this example it just so happened that the bounded candidate was bounded by a single competitor. This need not be the case—a bounding set can contain more than one candidate. Whenever a candidate has a bounding set that contains more than just one bounder, it is not enough to determine whether ❀CUMUL can remove some member of the bounding set. It needs to be determined whether ❀CUMUL can remove enough candidates from the bounding set that the remaining candidates cannot form a bounding set anymore. This is the topic of the next section.

4.2 Can ❀CUMUL remove enough candidates from a bounding set to break the bounds?

When the bounding set B(z) for some bounded candidate z contains more than one member, it is often the case that just removing a member of B(z) is not enough to liberate z from its bounding set. In the tableau below candidate z's maximal bounding set consists of three candidates, B(z) = {a, b, c}. However, two subsets of B(z) still qualify as bounding sets for z, namely {b}, {a, c}.

(12) **Multiple bounding sets for a single candidate**

	C_1	C_2
z	*	*
a		**
b		*
c	**	

Candidate z can be bounded by the set containing all three of candidates {a, b, c}. However, it is not necessary for all three to conspire to bound z. Any of the three sets {a, b, c}, {b} or {a, c} will suffice to bound z. In order to liberate z from its perpetual loser status, and to promote it to the set of possible winners, it is necessary to break all three of these sets. The minimal way in which this can be done, is to remove candidate b and at least one of a or c from B(z).

It is therefore not enough to simply remove any single candidate from a bounded candidate's maximal bounding set. To be sure that the candidate has been liberated from the power of its bounding set, it is necessary to check whether the remaining members of the maximal bounding still form a bounding set—that is whether they satisfy both the requirements on strictness and reciprocity. If the remaining candidates still qualify as a bounding set, then the bounded candidate is still bounded.

The general requirement that must be met for ❀CUMUL to liberate some bounded candidate from its bounding set, is therefore:

(13) **Requirements for ❀CUMUL to liberate a bounded candidate from the power of its bounding set**

Let z be a bounded candidate and B(z) its bounding set. Let ✯F be some faithfulness constraint that acts as a selector, and ❀*cand* a sympathetic candidate chosen by ✯F under some ranking of the constraints in CON.

Then the candidates that ❀CUMUL will remove from B(z) in concert with ❀*cand* form the potentially empty set ✯F-Remove(B(z)).

The residue of z's bounding set will then be Residue(B(z)) = B(z) - ✯F-Remove(B(z)).

For ❀CUMUL to liberate z from the power its bounding set, there must be at least one faithfulness constraint ✯F, and one sympathetic candidate ❀*cand* selected by ✯F such that the set Residue(B(z)) does not still qualify as a bounding set for z.

To determine whether ❀CUMUL complies with the requirements in (13) two things are necessary. First, it is necessary to know what all the possible sympathetic candidates are (since ❀CUMUL violations depend on the sympathetic candidate). Secondly it is necessary to know which candidates from B(z) will loose their membership of B(z) as a consequence of the violations assigned by ❀CUMUL. It has already been shown above in (9) that the candidates that ❀CUMUL (in concert with some sympathetic candidate) removes from B(z) can be determined in a ranking independent way. It has also been shown in (7) above that the set of potential sympathetic candidates can be determined in a ranking independent way. Both prerequisites are therefore met.

Once the list of potential sympathetic candidates has been compiled, it is possible to determine the violations marks that ❀CUMUL will assign with each of the possible sympathetic candidates. When the violation marks assigned by ❀CUMUL are known, the requirements in (9) above can be used to determine for each of the members of the bounding set B(z) whether ❀CUMUL will remove it from the bounding set. After all the members of B(z) that ❀CUMUL with a specific sympathetic candidate will remove from B(z) have been determined, the residue of candidates in B(z) can be determined—the candidates that are left after subtracting the candidates removed by ❀CUMUL from B(z). It can then be checked whether this residue of B(z) still qualifies as a bounding set for z. If one instance is found where the answer to this question is NO, then it has been determined that ❀CUMUL has liberated z from B(z).[11] However, if all of the possible sympathetic candidates have been exhausted and the answer to this question has been YES every time, then the conclusion is that ❀CUMUL was not able to liberate z from B(z).

This is illustrated now with an example based on (12), i.e. where B(z) = {a, b, c}, but where the subsets {b} and {a, c} also qualify as bounding sets for z. It is therefore necessary to remove minimally b and one of a or c from B(z) to free z from its bounding set. The tableau below represents a case like this.

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11 It has still not been shown that z is promoted to a potential winner. It is still necessary to show that there is some ranking of the constraints under which the sympathetic candidate under consideration will be chosen as sympathetic candidate, and under which z is better than the all its competitors. See the discussion in §4.3 on this issue.

(14) **❀CUMUL can free z from B(z)**

	M_1	M_2	F_1	F_2
z	*	*	*	*
a		**	*	
b		*	*	
c	**			*

When F_1 is designated as selector, candidate c will be chosen as sympathetic candidate. It is the only candidate satisfying this constraint, and therefore it is guaranteed to be chosen as sympathetic candidate. With ❀c as sympathetic candidate, both candidates a and b will loose their membership of B(z). This can be checked by using the requirements specified in (9). First, there is some faithfulness constraint in terms of which a does better than sympathetic ❀c (F_2(a) = 0 < F_2(❀c) = 1). Candidate b also satisfies constraint F_2, (i.e. F_2(b) = 0), and therefore b stands in the same relationship to ❀c as candidate a does. The first requirement in (9) is therefore satisfied with regard to both candidates a and b. The second requirement is that bounded candidate z should do at least as badly as sympathetic candidate ❀c on all faithfulness constraints. This is also the case (F_1(❀c) = 0 ≤ F_1(z) = 1, and F_2(❀c) = 1 ≤ F_2(z) = 1). The prediction is therefore that ❀CUMUL will remove both a and b from B(z) if ❀c is chosen as sympathetic candidate. This is confirmed by the tableau below.

(15) **❀CUMUL can free z from B(z)**

	M_1	M_2	✯F_1	F_2	❀CUMUL
z	*	*	*	*	✓
a		**	*		*
b		*	*		*
❀ c	**		✓	*	✓

With candidate ❀c as sympathetic candidate, ❀CUMUL removes both a and b from B(z). Only candidate ❀c remains and it alone does not count as bounding set for z—it satisfies the strictness, but not the reciprocity requirement. In this grammar, ❀CUMUL is therefore able to free z from the power of its bounding set.

However, if ❀CUMUL has been shown to remove enough members from B(z) such that z is no longer bounded, it has still not been shown that z can be a winner. The next section is dedicated to explaining why this is so, and to finding a solution to this problem.

4.3 Can the bounded candidate be optimal and the sympathetic candidate sympathetic under the same ranking conditions?

A selector constraint selects a set of candidates from which the sympathetic candidate must be chosen. All candidates obeying the selector constraint are contenders for the status of sympathetic candidate. The competition between these selector-obeying candidates is decided by the full constraint set excluding the selector constraint and the sympathetic constraints. The candidate in the set of selector-obeyers that is most harmonic in terms of the constraint set without the selector constraint and the sympathetic constraints is designated as the sympathetic candidate. If only one candidate were to obey the selector constraint, then its choice as sympathetic candidate is of course not dependent upon ranking at all.

However, it is not possible for there ever to be only one candidate satisfying the selector constraint. Since GEN supplies an infinite number of candidates, it is highly unlikely that there will ever be some constraint such that all candidates with the exception of one satisfy it. For the sake of argument accept that such a constraint can exist, and that the selector constraint is such a constraint—i.e. the selector is satisfied by only one candidate. The selector constraint is a faithfulness constraint. The fully faithful candidate is guaranteed to exist (Moreton 1999). From this it follows that the single candidate that obeys the selector constraint must be the fully faithful candidate. If the fully faithful candidate is chosen as the sympathetic candidate, then all other candidates will satisfy ❀CUMUL. Since the fully faithful candidate has no faithfulness violations, all unfaithful candidates will accumulate its faithfulness violations. ❀CUMUL will therefore assign no violations marks in a case like this, and will therefore not be able to break any bounding set. In any grammar where ❀CUMUL is able to break a bounding set, the sympathetic candidate must therefore be a candidate that is unfaithful on at least one constraint—see also (9a) on this. In such a case there will then always be at least two candidates obeying the selector constraint—the sympathetic candidate and the fully faithful candidate. Therefore, in all instances where ❀CUMUL can potentially break a bounding set, the choice of the sympathetic candidate will always depend on the ranking of the constraints.

After it has been shown that ❀CUMUL in concert with a specific sympathetic candidate can liberate some bounded candidate from its bounding set, it still needs to be shown that there is some ranking such that the sympathetic candidate is chosen as sympathetic candidate over all other candidates obeying the selector constraint, and such that this ranking does not conflict with the ranking required for the (previously) bounded candidate to be optimal. Only once this has been shown, can it be claimed that the bounded candidate has been promoted to the set of possible winners.

Below it will first be shown that this is a real problem. One example will be discussed where ❀CUMUL has broken the bounding set B(z) of bounded candidate z, but where z can still not be optimal. Then it will be shown how this problem can be solved in a ranking independent way.

4.3.1 Why it is not enough that ❀CUMUL breaks the bounds

Consider tableau (16) below. In this tableau candidate z is harmonically bounded by candidates a and b, i.e. B(z) = {a, b}. In this tableau the bounding set of z is broken by ❀CUMUL when candidate ❀c is chosen as sympathetic candidate:

(16) **Bounded candidate a loser even though bounding set has been broken**

	M_1	M_2	✭F_1	F_2	M_3	❀CUMUL
z	*	*	*	*		✓
a	**		✓	*		✓
b		**	*			*
❀ c		*	✓	*	*	✓
d			*	*	*	✓

When ✭F_1 acts as selector constraint, then both candidates a and ❀c are selected as potential sympathetic candidates. Since not one of a or ❀c is harmonically bounded in terms of the constraint set without the selector constraint ✭F_1 or the sympathy constraints, it follows that each of them will be selected as sympathetic candidate under at least one ranking. When candidate ❀c is chosen as sympathetic candidate, bounded candidate z is liberated from its bounding set—because b does not accumulate ❀c's faithfulness violations, while z does. (For an explanation of the conditions that must hold for this to happen see §4.1.) Candidate b therefore violates ❀CUMUL and candidate z does not. Candidate b consequently does not fulfil the reciprocity requirement for membership of B(z) anymore. But because b is not in B(z) anymore, candidate a also does not fulfil the reciprocity requirement anymore. Candidate z beats candidate a on M_1, and because b is not in B(z) anymore, candidate a cannot be saved by any member of

B(z). Candidate z has therefore been liberated from its bounding set—it has an empty bounding set.

But candidate z can still not be a winner. For z to be the winner, it is necessary that candidate ❀c be chosen as sympathetic candidate over candidate a (because z is liberated by the violations that ❀CUMUL assigns with reference ❀c). For ❀c to be chosen as sympathetic candidate over candidate a, it is necessary that the highest constraint in terms of which a and ❀c differ be one that favours ❀c over a. For z to be optimal it is also necessary that z beats sympathetic candidate ❀c, and this means that the highest ranked constraint in terms of which z and ❀c differ should be one that favours z over ❀c. Lastly, it also necessary that z beats d, and therefore that the highest ranked constraint in terms of which z and d differ be one that favours z over d. These requirements can be summarised as follows:

(17) **Requirements that must be met for z to be optimal**

a. $\mathbf{M_1} >> \mathbf{M_3}$, M_2

(This is to ensure that ❀c is chosen over a as sympathetic candidate.)

b. $\mathbf{M_3} >> \mathbf{M_1}$, ✭$F_1$

(This is to ensure that z beats sympathetic candidate ❀c.)

c. $\mathbf{M_3} >> \mathbf{M_1}$, M_2

(This is to ensure that z beats d.)

The requirements that must be met for ❀c to be chosen as sympathetic candidate ($M_1 >> M_3$) are in direct conflict with the requirements that must be met for z to beat ❀c, and for z to beat d ($M_3 >> M_1$). There is therefore no ranking possible under which ❀c will be chosen as sympathetic candidate and z will be chosen as optimal candidate. Since z's freedom from its bounding set depends on ❀c being chosen as sympathetic candidate it therefore follows that z cannot be optimal. Even though z has been liberated from its bounding set, it has still not been promoted to the set of possible winners.

This shows that even when ❀CUMUL liberates a bounded candidate from the domination of its bounding set, it is still not guaranteed that the (previously) bounded candidate is promoted to the set of potential winners. More is required—it is still necessary to show that there is at least one ranking in terms of which the relevant sympathetic candidate is chosen as sympathetic candidate, and in terms of which the bounded candidate is better than all other candidates. In the next section the requirements that must be met for this to be the case are determined. These requirements will be stated in a way that does not depend on ranking.

4.3.2 How to ensure that the bounded candidate is optimal and the sympathetic candidate is sympathetic

The source of the problem that was illustrated in the previous section is the competition between the bounded candidate z and the candidates not violating ❀CUMUL. It is possible that the ranking requirements for z to win these candidates are in conflict with the ranking requirements for the choice of the relevant sympathetic candidate. This problem does not exist for the candidates that violate ❀CUMUL. In the discussion below it will first be shown how the candidates violating ❀CUMUL can in a ranking independent way be distinguished from those obeying ❀CUMUL. Then it will be shown why the ❀CUMUL-violators do not cause problems. Finally, a solution will be proposed for the problems caused by the ❀CUMUL-obeyers.

The conditions that must be met for ❀CUMUL to remove $b \in B(z)$ from B(z) were stated in (9) (§4.1). Part of these conditions was that candidate b should violate ❀CUMUL. This part of (9) can generalised so that it applies to all candidates.

(18) **Conditions that must be met for any candidate to violate ❀CUMUL**

Let K be the set of all candidates, and $b \in K$. Let ❀c $\in K$ be a sympathetic candidate. Let $\mathbb{F}$ be the set of all faithfulness constraints. Candidate b violates ❀CUMUL iff:

$\exists F \in \mathbb{F}$ such that F(❀c) > F(b).[12]

❀CUMUL requires that the faithfulness violations of the candidate it evaluates form a superset of the faithfulness violations of the sympathetic candidate. If there exists a faithfulness constraint in terms of which some candidate earns fewer violations than the sympathetic candidate, then this requirement is not satisfied for that candidate. This is what is captured in (18). Note that there is no reference to ranking in (18). It is therefore possible to determine the ❀CUMUL-violators without recourse to ranking between constraints.

Now that it has been shown that the candidate set can be divided into two sets, the ❀CUMUL-obeyers and ❀CUMUL-violators, the competition between the bounded candidate and each of these two sets will be considered separately. First, consider the set of ❀CUMUL-violators. For ❀CUMUL to liberate candidate z from its bounding set B(z), it is necessary that z not violate ❀CUMUL (see §4.1). Since the ranking of ❀CUMUL is not relevant in the determination of the sympathetic

12 Where F(x) indicates the number of violations constraint F assigns to candidate x.

candidate, it is possible to rank ❀CUMUL in the top layer of constraints without affecting the choice of sympathetic candidate. Since candidate z does not violate ❀CUMUL, it is therefore possible for z to be better than all candidates that violate this constraint without this having any influence on the choice of the sympathetic candidate. No candidate from the set of ❀CUMUL-violators can therefore cause the type of problems illustrated in the previous section (§4.3.1).

The real problem comes from the set of ❀CUMUL-obeyers.[13] Looking back at tableau (16) it will be clear that it was exactly these candidates that caused the problems. (The references to candidates below are to the candidates in tableau (16)). Candidate b (the ❀CUMUL-violator) was no problem. It was d and ❀c, both of which obey ❀CUMUL, that prevented z from being chosen as optimal. Even though both d and ❀c obey ❀CUMUL, they do differ in one important respect. Candidate ❀c obeys the selector constraint ✯F_1, while candidate d does not. It is therefore possible to divide the set of candidates that obey ❀CUMUL into two smaller groups, those that also obey the selector and those that violate the selector.

(19) **The set of ❀CUMUL-obeyers**

(a) ❀CUMUL-obeyers also obeying selector constraint (candidate ❀c).

(b) ❀CUMUL-obeyers that violate the selector constraint (candidate d).

Sympathetic candidate ❀c is of course also in the set of candidates that obey both the selector and ❀CUMUL—i.e. in set (19a). Also, since ❀c as sympathetic candidate must beat all candidates that obey the selector constraint, it follows that it must be the most harmonic of all the candidates in (19a). If it can therefore be shown that z beats ❀c, it has been shown that z beats all the candidates in (19a). All that then needs to be shown is that z beats d—i.e. the candidates in (19b). In terms of the candidates in tableau (16) the following is what has to be shown:

13 The bounded candidate z is also a member of this group (see §4.1). However, a candidate cannot compete with itself. All references to the set of ❀CUMUL-obeyers therefore exclude the bounded candidate.

(20) **Conditions that must be met for z to be optimal**

(a) **❀c** ≻ **a** (The correct sympathetic candidate is chosen.)

(b) **z** ≻ **❀c** (Bounded candidate beats the candidates that obey both ❀CUMUL and the selector.)

(c) **z** ≻ **d** (Bounded candidate beats the candidates that obey ❀CUMUL, but violate the selector.)

To impose these three harmonic orderings on the candidate set at most three constraints are needed—one for each of the three orderings.[14]

(21) **Constraints required to impose the harmonic ordering in (20)**

C_1(z) < C_1(❀c) C_2(❀c) < C_2(a) C_3(z) < C_3(d)

But the mere existence of three such constraints is still not enough. One of these constraints must outrank the other two. This constraint cannot conflict with the other two constraints on the orderings that they have to impose. Similarly, one of the constraints must be ranked in second position. This constraint can then not conflict with the third constraint on the ordering that it has to contribute. At least one of the three constraints must therefore not conflict with either of the other two. And of the remaining two constraints at least one must not conflict with the last constraint.

The required situation is illustrated in tableau (22) below. In this tableau below C_1 imposes the ordering **z ≻ ❀c** on the candidate set, but is quiet about the ordering between z and d, and between ❀c and a. This constraint can therefore be ranked in highest position. C_2 imposes the ordering **❀c ≻ a**. It also conflicts with the ordering imposed by C_1 (i.e. it orders **❀c ≻ z**). This does not matter since C_1 outranks C_2. C_2 is silent about the ordering between z and d, and C_2 can therefore dominate C_3, which will be responsible for establishing the desired ordering between these two candidates. C_3 imposes the ordering **z ≻ d**. C_3 also conflicts with both C_1 and C_2 on the crucial orderings imposed by them. However, since C_3 is ranked lower than these constraints, the fact that it conflicts with them does not matter.

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14 This is "at most", because it is possible that a single constraint can be responsible for imposing more than one of these orderings.

(22) **Imposing the ordering z ≻ ❀c ≻ a and z ≻ d on the candidate set**

	C1	C2	C3
z		*	**
a	*	*	
❀ c	*		*
d		*	***

In tableau (16) sympathetic candidate ❀c has only one competitor for sympathetic status, candidate a. There is also one candidate in the set of candidates that obey ❀CUMUL but violate the selector, namely candidate d. It is, of course, possible that both of these sets can contain multiple candidates. The requirements above must therefore be generalised to apply to all candidates in both of these sets. This is done in (23) below. Note that in the formulation of these conditions no reference is made to ranking. The conditions are stated in terms of the existence of certain constraints.

(23) **Conditions (i) for the sympathetic candidate to be chosen as sympathetic candidate, (ii) for the bounded candidate to be better than the sympathetic candidate, (iii) and for the bounded candidate to be better than all candidates obeying ❀CUMUL but violating the selector constraint**

Let Σ *be* the set of all constraints (excluding the sympathy constraints) and ✩F the selector constraint.

Then $\text{Cand}_{✓✩\text{F}}$ is the set of candidates obeying ✩F, and $\text{Cand}_{×✩\text{F}}$ is the set of candidates violating ✩F.

Let $\text{Cand}_{✓❀\text{CUMUL}}$ be the set of candidates obeying ❀CUMUL.

Then $\text{Cand}_{×✩\text{F}\ \&\ ✓❀\text{CUMUL}} = \text{Cand}_{×✩\text{F}} \cap \text{Cand}_{✓❀\text{CUMUL}}$, the set of constraints obeying ❀CUMUL but violating the selector constraint.

Let *bound* be a bounded candidate and B(*bound*) its maximal bounding set.

Let *❀cand* be the candidate from $\text{Cand}_{✓✩\text{F}}$ that must be chosen as sympathetic candidate to liberate *bound* form its bounding set B(*bound*).

Then the set of candidates competing with *❀cand* to be chosen as sympathetic candidate is $\text{Cand}_{✓✩\text{F-❀}} = \text{Cand}_{✓✩\text{F}}$ - *❀cand*.

There is a ranking between the constraints in Σ in terms of which *bound* will be better than any candidate in the set Cand$_{\times\star F\ \&\ \checkmark❀CUMUL}$ and better than ❀*cand*, and in terms of which ❀*cand* will be chosen as sympathetic candidate iff:

For ∀ *cand*$_{\times\star\&\checkmark❀}$ ∈ Cand$_{\times\star F\ \&\ \checkmark❀CUMUL}$, and for ∀ *cand*$_{\checkmark\star\text{-}❀}$ ∈ Cand$_{\checkmark\star F\text{-}❀}$,

∃ C_1, C_2, C_3 ∈ Σ, such that C_1, C_2, C_3 are not necessarily distinct, and such that

[1] C_1(*bound*) < C_1(❀*cand*)

AND

[2] C_2(❀*cand*) < C_2(*cand*$_{\checkmark\star\text{-}❀}$)

AND

[3] C_3(*bound*) < C_3(*cand*$_{\times\star\&\checkmark❀}$)

AND

[4] Any one of the of the following:

(a) C_1 does not conflict with C_2 or C_3, and C_2 does not conflict with C_3.

(b) C_1 does not conflict with C_2 or C_3, and C_3 does not conflict with C_2.

(c) C_2 does not conflict with C_1 or C_3, and C_3 does not conflict with C_1.

(d) C_2 does not conflict with C_1 or C_3, and C_1 does not conflict with C_3.

(e) C_3 does not conflict with C_1 or C_2, and C_1 does not conflict with C_2.

(f) C_3 does not conflict with C_1 or C_2, and C_2 does not conflict with C_1.

Now reconsider the competition between bounded candidate z, and its competitors d and ❀c in tableau (16). The tableau is repeated here for easier reference.

(24) **Bounded candidate a loser even though bounding set has been broken**

	M_1	M_2	✯F_1	F_2	M_3	❀CUMUL
z	*	*	*	*		✓
a	**		✓	*		✓
b		**	*			*
❀ c		*	✓	*	*	✓
d			*	*	*	✓

It is now possible to see in terms of the conditions in (23) why candidate z can never be a winner even though it is liberated from its bounding set. In this tableau, clauses [1], [2] and [3] of (23) are satisfied, but not clause [4]. Clause [1] requires that there be a constraint that favours the bounded candidate z over the sympathetic candidate ❀c—M_3 does this. Clause [2] requires that there be a constraint that favours sympathetic candidate ❀c over candidate a (the candidate that competes with it for sympathetic status)—M_1 does this. Clause [3] is then also satisfied by M_3, since bounded candidate z does better than candidate d (as the candidate obeying ❀CUMUL, but violating the selector). Clauses [1], [2] and [3] are therefore satisfied by the two constraints M_1 and M_3. Clause [4] then requires *inter alia* that at least one of the constraints that satisfy the first three clauses not conflict with the other on the crucial orderings. To see that clause [4] cannot be satisfied, consider the following table:

(25) **Why clause [4] cannot be satisfied**

Constraint	Crucial ordering	Conflicts with
M_1	❀c $\succ$ a	M_3 (❀c, d $\succ$ z)
M_3	z $\succ$ ❀c,d	M_1 (a $\succ$ ❀c)

It is therefore now possible, without recourse to ranking, to determine that candidate z cannot be optimal. Because clause (23)[4] is not satisfied in tableau (24) it is correctly predicted that even though z has been liberated from its bounding set, it can still not be optimal.

Candidate z can be promoted to the set of possible winners in several ways. One of these ways are used here as an example. The tableau below has the same candidates as tableau (24), and also all of the constraints that were in (24) with the same violations assigned to the candidates by each of these constraints. However, the tableau below has two additional constraints that were not in (24), namely M_5 and M_6.[15] In the tableau below, the constraints are also ranked—unlike in (24) above. Although the constraints therefore occur in a different order from what they did in (24) their names have been kept the same in order to make comparison easier.

(26) **Bounded candidate z finally wins!**

	❀CUMUL	M_6	M_5	M_1	M_3	M_2	✮F_1	F_2
☞ z			*¡	*		*	*	*
a			**!	**			✓	*
☜ b	*!					**	*	
❀ c			**!		*	*	✓	*
d		*!			*		*	*

How does this tableau do in terms of the requirements in (23)? If those requirements are correct, they should predict that z will be promoted to the set of possible winners in this tableau—which z obviously is; in fact, it is the winner in this tableau. Clause (23)[1] requires that there be a constraint that favours the bounded candidate z over the sympathetic candidate ❀c—the new constraint M_5 does this. Clause [2] requires that there be a constraint that favours sympathetic candidate ❀c over candidate a (the candidate that competes with it for sympathetic status)—M_1 still does this. Clause [3] is then satisfied by new constraint M_6, since bounded candidate z does better than candidate d on M_6. All that then remains is to check whether this tableau also satisfies clause [4]. Consider the table below:

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15 It is important to check that the addition of these two constraints still leaves z's bounding set B(z) = {a, b} in tact. M_6 obviously has no influence on the bounding set—not bounded candidate z nor any of the members of its bounding set B(z) violates this M_6. Constraint M_5 is potentially relevant. In terms of this constraint bounded candidate z beats candidate a, one of its bounders. However, the other member of B(z), candidate b, in turn beats bounded candidate z on M_5. Reciprocity is therefore still satisfied, and the bounding set B(z) is still in tact.

(27) **To show that (25) satisfies (23)[4]**

Constraint in (26)	Constraint in (23)	Crucial ordering	Conflicts with
M_5	C_1	z ≻ ❀c	M_6 (d ≻z)
M_1	C_2	❀c ≻ a	M_5 (❀c≻z)
			M_6 (d ≻z)
M_6	C_3	z ≻ d	—

In terms of the constraints as they are referred to in (23), it is therefore true that C_3 does not contradict either C_1 or C_2, and that C_1 does not contradict C_2. This is what is required in clause [4](e). Tableau (26) therefore satisfies all four requirements stated in (23), and is correctly predicted that bounded candidate z in this tableau is promoted to the set of possible winners.

4.4 Conclusions about the optimal status of the bounded candidate and the choice of the sympathetic candidate

In this section of the paper the conditions that must be met for ❀CUMUL to promote a bounded candidate z to the set of potential winners have been determined. This was done incrementally as follows:

(28) **Section §4 determined the conditions that must be met for:**

a. ❀CUMUL to remove candidates from the bounding set B(z) of some bounded candidate z—§4.1 (9).

b. ❀CUMUL to remove enough members from the bounding set B(z) that z is no longer bounded—§4.2 (13).

c. Sympathetic candidate ❀c to be better than all candidates competing with it to be chosen as sympathetic candidate, and bounded candidate z to be better than all of its competitors—§4.3.2 (23).

The conditions that were stated made no reference to ranking. Therefore, it follows that it is possible to determine the candidates that ❀CUMUL can add to the set of potential winners in a ranking independent way.

5. The set of potential winners is still finite

This section will show that the set of potential winners is still finite, even after the candidates that ❀CUMUL can promote from perpetual losers to potential winners have been added to the set of potential winners. The argument consists of two steps. The fist shows that for any given input the number of possible sympathetic candidates is finite. The second step then shows that with reference to a single sympathetic candidate, ❀CUMUL can promote only a finite number of bounded candidates to the set of potential winners. The argument makes crucial use of Samek-Lodovici and Prince's result that the set of non-harmonically bounded candidates in an infinite candidate set, is finite in number.

It can be shown that for any given input, a selector constraint can select only a finite number of sympathetic candidates. The candidates that compete for sympathetic status are all those candidates from the full candidate set that obey the selector constraint. This set of selector-obeyers can be infinite in number. However, only those candidates from this set that are not harmonically bounded by the other members in this set, are potential sympathetic candidates – see (7) in §4 on this. Here Samek-Lodovici and Prince's result can be used. They have shown that the set of non-harmonically bounded candidates from an infinite candidate set, is finite in number. Although the set of candidates selected by every potential selector constraint can be infinite in number, the subset of each of these sets that are actually possible sympathetic candidates, is finite in number.

Under the assumption that the constraint set CON is finite in number, it follows that the set of potential selector constraints (the faithfulness constraints), is also finite. There is therefore a finite number of selector constraints, each of which selects only a finite number of potential sympathetic candidates. From this it follows that for any given input, there is only a finite number of potential sympathetic candidates.

The second part of the argument concerns the number of bounded candidates that ❀CUMUL with reference to a specific sympathetic candidate can promote to the set of potential winners. For some bounded candidate z to be liberated from its bounding set B(z) by ❀CUMUL, it is necessary that z not violate ❀CUMUL – see (9) in §4.1 on this requirement. Any candidate liberated from its bounding set by ❀CUMUL, must therefore be in the set of ❀CUMUL-obeyers. However, this set is infinite in number. The full candidate set is infinite in number, and dividing it into the two disjoint subsets of ❀CUMUL-obeyers and ❀CUMUL-violators, simply results in two smaller but still infinite sets. Limiting the set of bounded candidates that ❀CUMUL can promote to potential winners to the set of ❀CUMUL-obeyers, does not solve the infinity problem. But, not all candidates in the set of ❀CUMUL-obeyers are potential winners. Only those candidates in this set that are not bounded in this set, are potential winners. Here Samek-Lodovici and Prince's result come in handy again. Although the set of ❀CUMUL-obeyers is infinite in number, the subset of this set that constitutes the potential winners, is finite in number. Therefore, with reference to any sympathetic candidate, ❀CUMUL can promote only a finite number of bounded candidates to the set of potential winners.[16]

This establishes the result. For any given input, there is only a finite number of possible sympathetic candidates. With reference to each of these sympathetic candidates, ❀CUMUL can promote only a finite number of bounded candidates to the set of potential winners. The set of potential winners excluding all harmonically bounded candidates is finite in number (this is Samek-Lodovici and Prince's result). Adding a finite number of finite sets to a finite set, results in a larger but still finite set. Even in an OT grammar with ❀CUMUL, the set of potential winners is still a finite set.

6. Summary, conclusion, unresolved issues

Two of the most important results of Samek-Lodovici and Prince's paper are:

(29) **Central results of Samek-Lodovici and Prince**

a. That the set of candidates that are potential winners is finite in number.

b. That this set of candidates can be determined in a ranking independent way.

The candidates that are perpetual losers, are all candidates that can never be optimal under any ranking of the constraints, i.e. they are harmonically bounded candidates. However, in non-surface apparent/counter-feeding opacity, a harmonically bounded candidate has to be optimal. A candidate that falls into the group of perpetual losers therefore has to be promoted to the set of potential winners. Sympathy Theory was developed as a way to deal with this problem within OT. Through the sympathy constraints ❀CUMUL and ❀DIFF, it is possible to promote certain harmonically bounded candidates from the set of perpetual losers to the set of potential winners. This paper represents an investigation into the influence of Sympathy Theory on the results of Samek-Lodovici and Prince.

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16 This is the worst possible scenario. Not all of the non-harmonically bounded candidates in the set of ❀CUMUL-obeyers are candidates that ❀CUMUL can promote from being perpetual losers to being potential winners. Some of these candidates are potential winners even without sympathetic constraint interaction. There are also many of these candidates that ❀CUMUL can liberate from their bounding sets, but that can still never be optimal—see §4.3. This section therefore does not really determine that the set of candidates that ❀CUMUL can promote from being perpetual losers to being potential winners is finite in number. Rather, it shows that some superset of this set is finite.

The result in (29b) will hold only if it can be shown that all harmonically bounded candidates that can be promoted to the set of potential winners by any of the two sympathy constraints, can be determined without recourse to ranking. In this paper it was shown that this is true of the constraint ❀CUMUL. It still needs to be shown that the same is true of the constraint ❀DIFF. Since it is possible to determine all candidates that are potential sympathetic candidates in a ranking independent way,[17] and since the violations assigned by ❀CUMUL and ❀DIFF can be determined when the sympathetic candidate is known, it will be possible to get the same results for ❀DIFF that were attained for ❀CUMUL in this paper.

One more cautionary note should be added at this point. In this paper it was assumed that there can be only one selector constraint and therefore only one sympathetic candidate per grammar (see §3.1). McCarthy (1999) has shown that this is not the case, and that it is sometimes required to have more than one selector constraint, more than one sympathetic candidate, and therefore more than one set of sympathy constraints. The conclusions reached in this paper about a sympathy grammar with only one selector constraint, can be adapted to be true of a sympathy grammar with any number of selector constraints. For a grammar with two selector constraints, it has to be determined which members of a bounded candidate z's bounding set B(z) can be eliminated by each of the two sets of sympathy constraints. This can be done straightforwardly in the ways explained in this paper. Then it can be checked whether the residue of the bounding set B(z) still qualifies as a bounding in the regular way. There is one complication however—it needs to be checked that the ranking conditions that must hold for the candidate chosen as sympathetic candidate by the first selector, and the ranking conditions that must hold for the candidate chosen as sympathetic candidate by the second selector, do not conflict. If they conflict, then they can of course not be chosen as sympathetic candidates under the same ranking, and therefore they cannot remove members from B(z) simultaneously. In this paper it has been illustrated how it can be determined whether two ranking conditions conflict or not without actually referring to the ranking between constraints. It should therefore be possible to also resolve this question in a similar non-ranking dependent manner.

Consequently, it can be concluded with some confidence that Samek-Lodovici and Prince's result (29b) is also true of an OT grammar that allows ❀CUMUL and ❀DIFF, and more than one selector constraint.

Once it has been shown that (29b) hold, it needs to be shown that the set of potential winners is still finite (29a). This is also easily done for an OT

17 See (7) in §4.

grammar with ❀CUMUL. For any given input the number of possible sympathetic candidates is finite, and the number of candidates that ❀CUMUL can promote from perpetual losers to potential winners with reference to any specific sympathetic candidate is finite. Therefore, even in an OT grammar with ❀CUMUL, the set of potential winners is still finite in number. As before, it has not been shown that this result is also true once ❀DIFF is added into the mix, or if more than one selector constraint per grammar is allowed. However, by way of reasoning similar to that in this paper, it will be possible to show that this result is also true even under these circumstances.[18]

In conclusion, it has been shown that Samek-Lodovici and Prince's results are also true of an OT grammar that allows sympathetic constraints. This is an important result. Adding constraints with global vision (like sympathy constraints) to the grammar, obviously adds much power to the grammar. If OT is to account successfully for phenomena like non-surface apparent/counter-feeding opacity, Sympathy Theory (or something very similar to it) is required. The grammar can therefore not do without the additional power. However, at the same time it is important that the grammar not become too powerful. This paper has shown that even though Sympathy Theory does argue for a substantially more powerful grammar, the grammar is still rather constrained. It does not negate the very appealing features of a classic OT grammar that Samek-Lodovici and Prince have identified.

The fact that OT lends itself to formal analyses such as in this paper, should be exploited more. Analyses such as these lend themselves to testing the predictions and implications of the theory on a theoretical and formal level. They might show potential strengths and weaknesses in the theory that are not easily visible in more data-oriented studies.

18 For a development of this line of reasoning, see Coetzee (2003). In that paper it is shown that the set of potential winners is finite even in the following situations: (i) when multiple selectors are allowed, (ii) when both ❀CUMUL and ❀CUMUL are allowed, (iii) when sympathetic constraint interaction is mediated by ordinary correspondence constraints rather than cumulative constraints, and (iv) when OO-Correspondence constraints is added to CON.

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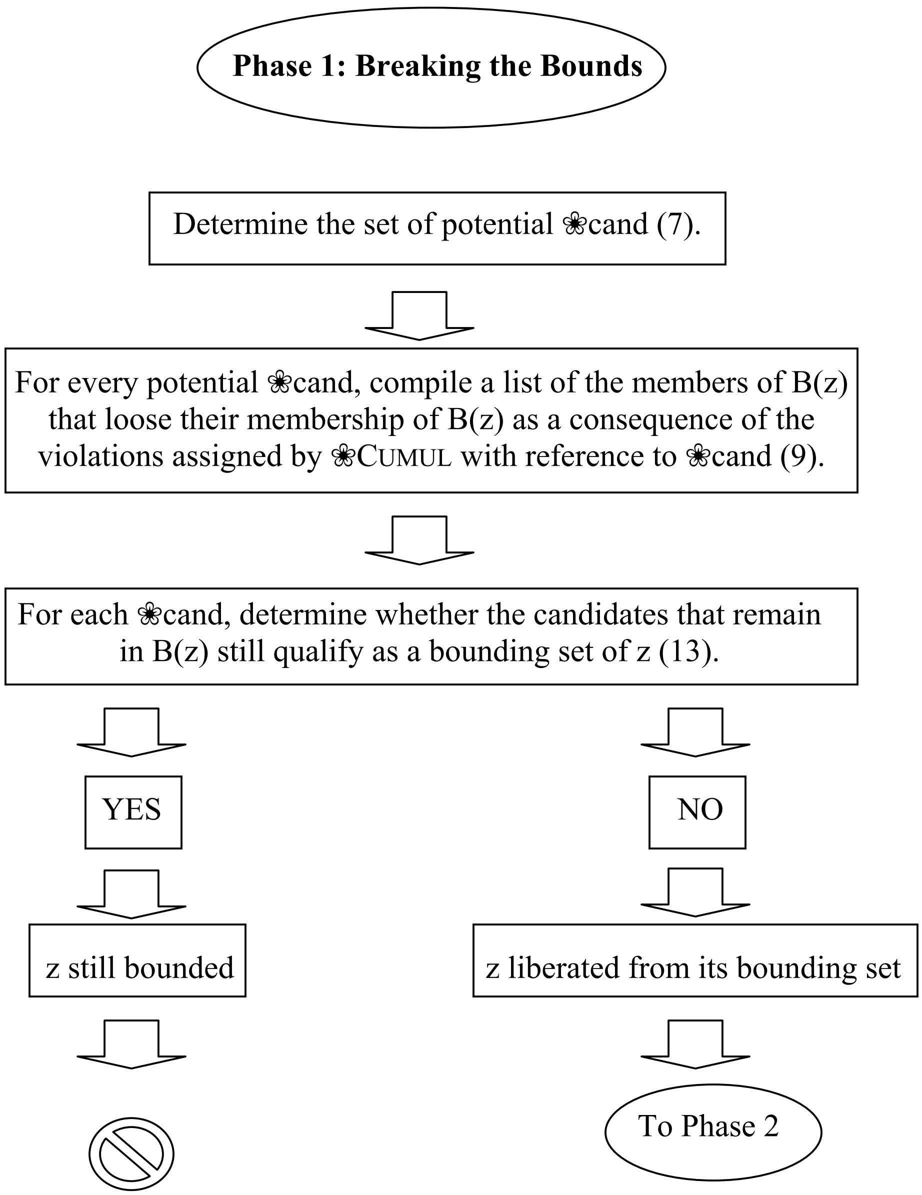
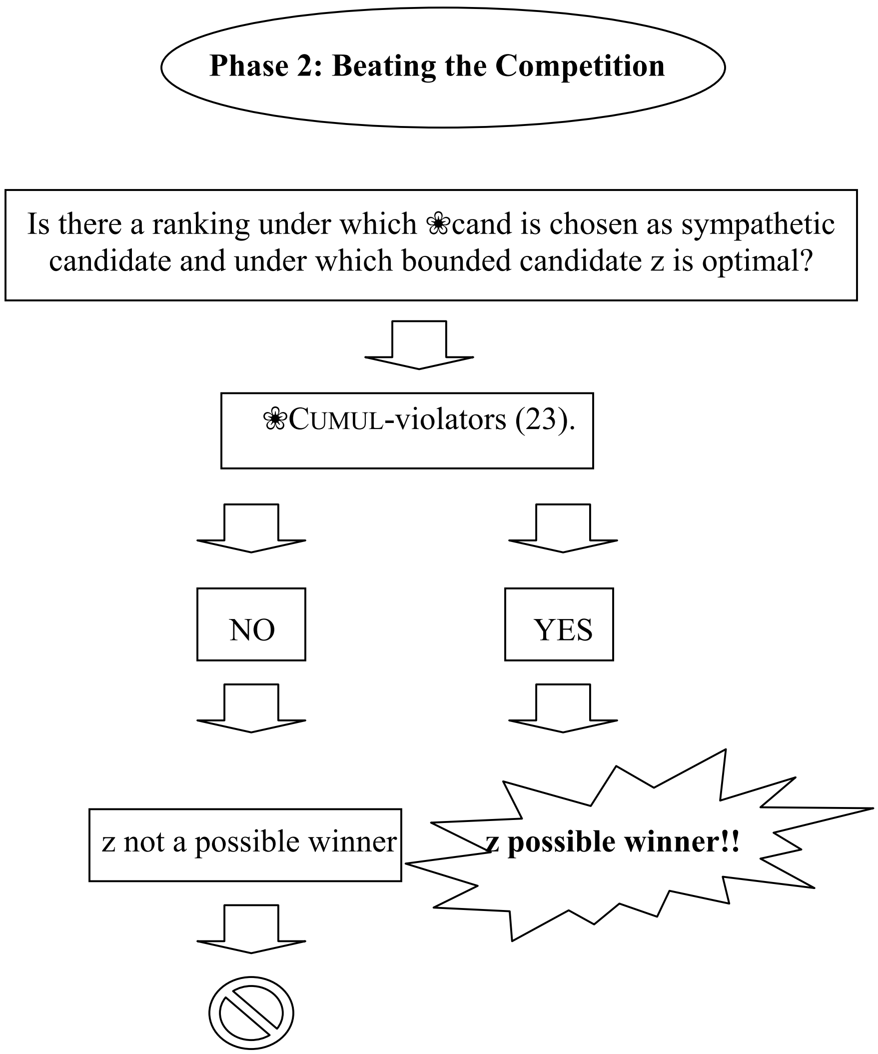
Appendix A: Flowchart of the argument

How to determine whether ❀CUMUL promotes some harmonically bounded candidate z from the set of perpetual losers to the set of potential winners

Let z be a bounded candidate and B(z) its bounding set. The following diagram shows the steps to go through in order to determine whether ❀CUMUL can promote z from the set of perpetual losers to the set of potential winners.




Phase 2: Beating the Competition
Is there a ranking under which ❀cand is chosen as sympathetic candidate and under which bounded candidate z is optimal?
❀CUMUL-violators (23).
NO
YES
z not a possible winner
z possible winner!!