A note on the construction of complex OT constraints by material implication*

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Smolensky (1995) proposes that Optimality Theory (Prince & Smolensky 2004 [1993]) be augmented with a mechanism of local constraint conjunction to permit the construction of complex constraints from simpler ones. A conjoined constraint $[A\&B]_D$ is violated if and only if constraint A and constraint B are both violated within some domain D.

While local conjunction has come to enjoy widespread, though far from universal, acceptance among practitioners of OT, less attention has been paid to the possibility that there might be other mechanisms for building complex constraints from simpler ones. This possibility was first advocated in Crowhurst and Hewitt (1997). They note that—if constraint-satisfaction is treated as equivalent to truth and constraint-violation as equivalent to falsehood—the semantics of local conjunction are identical to those of classical logical *dis*junction. The disjunction of two propositions is false if and only if both of the propositions are false—just as a locally conjoined constraint is violated iff both of its conjuncts are violated. (However, local conjunction is equivalent to classical conjunction if constraint-violation is equated with truth and satisfaction with falsehood—a conjunction of two propositions is true iff both conjuncts are satisfied, and likewise a locally-conjoined constraint is violated iff both conjuncts are violated.)

Having observed the parallel between local conjunction and logical disjunction, Crowhurst and Hewitt (1997) go on to suggest that classical conjunction and material implication also are needed as connectives for building complex OT constraints. This squib is concerned with one empirical consequence of admitting material implication as a tool for constraint-building. The classical semantics for material implication are given in the truth table below, together with the semantics of a complex constraint constructed by material implication, under the assumption that violation is equivalent to falsehood and satisfaction to truth:

(1)

P	Q	P→Q	P	Q	P→Q
0	0	1	*	*	✓
1	0	0	✓	*	*
0	1	1	*	✓	✓
1	1	1	✓	✓	✓

As shown, the material implication P→Q is false iff P is true and Q is false. Importantly, though, if P is false, P→Q is true regardless of whether Q is true or false. Likewise, a constraint P→Q will be violated only just in case constraint P is satisfied and constraint

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Q is violated. If constraint P is violated, P→Q will be satisfied irrespective of whether Q is violated or not.

The semantics of Crowhurst & Hewitt's (1997) proposed material implication connective for OT in fact differ from this—they treat the constraint [A→B] as satisfied iff A and B are both satisfied. However, classical material implication as a tool for building complex constraints is proposed by Archangeli, Moll, & Ohno (1998) and Balari, Marín, & Vallverdú (2000)—the latter in reply to Crowhurst & Hewitt (1997). For these authors, a constraint [A→B] is always satisfied by a given candidate if that candidate violates constraint A, since a material implication, as mentioned, is always true if its antecedent is false.

Allowing complex OT constraints to be constructed using a connective with these semantics has an interesting and almost certainly undesirable consequence: if A is a faithfulness constraint and B a markedness constraint, the constraint [A→B] will be able to function as an input-output antifaithfulness constraint, permitting OT to model circular chain shifts. To illustrate how this can come about, suppose that we have two binary distinctive features [±α] and [±β], and the following constraint ranking:

(2)
IDENT[+α] » [IDENT[-β] → *[+α]] » *[+β] » IDENT[-β], *[+α]

Constraint definitions:
IDENT[+α]: Assign a violation-mark if an input [+α] segment corresponds to an output [-α] segment.
IDENT[-β]: Assign a violation-mark if an input [-β] segment corresponds to an output [+β] segment.
*[+β]: Assign a violation-mark for every [+β] segment in the output.
*[+α]: Assign a violation-mark for every [+α] segment in the output.

Given the ranking in (2), consider what happens to an input segment that is [+α, +β]:

(3)

[+α, +β]	IDENT[+α]	[IDENT[-β]→ *[+α]]	*[+β]	IDENT[-β]	*[+α]
a. [+α, +β]		*	*!		*
b.► [+α, -β]		*			*
c. [-α, +β]	*!		*		
d. [-α, -β]	*!				

The undominated constraint IDENT[+α] rules out all candidates that change the input's [+α] to [-α]. This leaves [+α, +β] and [+α, -β] as contenders. The complex constraint [IDENT[-β] → *[+α]] will not distinguish between these two candidates: first, since the input does not contain a feature specification [-β], the antecedent IDENT[-β] is vacuously satisfied by all candidates; and second, both of the remaining contenders are [+α], and therefore violate the consequent of the conditional. Hence, both of the remaining contenders equally violate [IDENT[-β]→ *[+α]], by virtue of satisfying the antecedent while violating the consequent. The choice is then passed down to the markedness constraint *[+β], which chooses [+α, -β] over [+α, +β].

Now consider what happens when the input is [+α, -β]:

(4)

[+α, -β]	IDENT[+α]	[IDENT[-β] → *[+α]]	*[+β]	IDENT[-β]	*[+α]
a. ►[+α, +β]			*	*	*
b. [+α, -β]		*!			*
c. [-α, +β]	*!		*	*	
d. [-α, -β]	*!				

As before, the undominated status of IDENT[+α] immediately reduces the set of contenders to [+α, +β] and [+α, -β]. Moving down to the next highest-ranked constraint, we now encounter [IDENT[-β] → *[+α]]. Unlike before, the input now contains a [-β] specification, so it is no longer the case that all candidates vacuously satisfy the antecedent of the conditional.

The faithful candidate [+α, -β] satisfies the antecedent (IDENT[-β]) by virtue of preserving the input's [-β] specification, but it violates the consequent (*[+α]) by virtue of having a [+α] specification. Because it satisfies the antecedent but violates the consequent, this candidate violates the material implication. By contrast, the unfaithful candidate [+α, +β] also violates the consequent *[+α], as it too has an output [+α] specification, but it differs from the faithful candidate in violating the antecedent constraint IDENT[-β]. Because it violates the antecedent, [+α, +β] satisfies the material implication, and thus wins.

Building complex constraints by material implication thus allows us to model circular chain shifts in OT. In our example, input [+α, +β] surfaces as [+α, -β], while input [+α, -β] surfaces as [+α, +β]. The reason for this has to do with the fact that our complex constraint [IDENT[-β] → *[+α]] has a faithfulness constraint as its antecedent and a markedness constraint as its consequent. Such a constraint rewards unfaithfulness, as can be seen in (4): [+α, +β] violates the antecedent by being unfaithful, and by so doing it is exempted from having to satisfy the consequent, whereas the faithful candidate [+α, -β] satisfies the antecedent, and therefore would have to satisfy the consequent (which it doesn't) in order to satisfy the material implication. So, even though, for input /[+α, -β]/, faithful [+α, -β] and unfaithful [+α, +β] both violate *[+α], only the faithful candidate violates the material implication [IDENT[-β] → *[+α]].

Moreton (1996) has given a formal proof that OT is unable to model circular chain shifts. This proof rests on the assumption that OT grammars contain only two kinds of constraints: markedness constraints (defined as constraints which always assign the same number of violations to a given output form, regardless of what the input might be) and faithfulness constraints (defined as constraints which always assign zero marks to a candidate which is identical to the input).

Inspection of tableaux (3)-(4) quickly reveals that [IDENT[-β] → *[+α]] is neither a markedness constraint nor a faithfulness constraint, under these definitions. It isn't a markedness constraint, since the candidate [+α, +β] gets a violation from the complex constraint when the input is /[+α, +β]/, as in (3), but does not get a violation when the input is /[+α, -β]/, as in (4). Nor is [IDENT[-β] → *[+α]] a faithfulness constraint, since the fully-faithful candidates (3)a and (4)b both incur violations from it. Instead, given the appropriate circumstances in (4), [IDENT[-β] → *[+α]] can serve as an input-output *anti*faithfulness constraint: it prefers an unfaithful candidate over a faithful one, because being unfaithful (violating the antecedent) exempts a candidate from a pressure to be unmarked (i.e., to obey the consequent).

While circular chain shifts do seem to arise in morpheme realization and in tone sandhi, it is arguably the case that no language has input-output exchange processes that take place for strictly phonological (as opposed to morphologically- or syntactically-influenced) reasons (Anderson & Browne 1973, McCawley 1974, Moreton 1996, Alderete 1999, Wolf 2006). Whereas classical OT with only markedness and faithfulness constraints derives this empirical generalization as a formal universal, allowing the creation of complex constraints by material implication would subvert that result. This suggests that proposals for allowing such constraint-building are to be eschewed.

References

Alderete, John D. (1999). *Morphologically Governed Accent in Optimality Theory*. Ph.D. dissertation, University of Massachusetts, Amherst. [ROA-309]

Anderson, Stephen R., and Wayles Browne (1973). On keeping exchange rules in Czech. *Papers in Linguistics* **6**, pp. 445-482.

Archangeli, Diana, Laura Moll, and Kazutoshi Ohno (1998). Why not *NÇ̥. In M. Catherine Guber, Derrick Higgins, Kenneth S. Olson, and Tamara Wysocki (eds.), *CLS 34, Part 1: The Main Session*. Chicago: CLS, pp. 1-26

Balari, Sergio, Rafael Marín, and Teresa Vallverdú (2000). Implicational constraints, defaults, and markedness. GGT Report de Recerca GGT-00-8, Universitat Autònoma de Barcelona. [ROA-396]

Crowhurst, Megan, and Mark Hewitt (1997). Boolean operations and constraint interactions in Optimality Theory. Ms., University of North Carolina, Chapel Hill, and Brandeis University, Waltham, MA. [ROA-229]

McCawley, James D. (1974). [Review of *The Sound Pattern of English*]. *International Journal of American Linguistics* **40**, pp. 50-88.

Moreton, Elliot (1996). Non-computable functions in Optimality Theory. Ms., University of Massachusetts, Amherst. [ROA-364]

Prince, Alan, and Paul Smolensky (2004 [1993]). *Optimality Theory: Constraint Interaction in Generative Grammar*. Oxford: Blackwell. [ROA-537]

Smolensky, Paul (1995). On the structure of the constraint component CON of UG. Talk presented at University of California, Los Angeles. [ROA-86]

Wolf, Matthew (2006). For an autosegmental theory of mutation. In Leah Bateman, Adam Werle, Michael O'Keefe, and Ehren Reilly (eds.), *University of Massachusetts Occasional Papers in Linguistics* **32**: *Papers in Optimality Theory III*. Amherst: GLSA. [ROA-754]