

Review of Smolensky and Legendre, *The Harmonic Mind**

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April 13, 2009

Paul Smolensky & Géraldine Legendre (2006) *The Harmonic Mind: From Neural Computation to Optimality-Theoretic Grammar*. Cambridge, Mass.: MIT Press. Volume 1: *Cognitive Architecture*, Pp. xvii+563. Volume 2: *Linguistic and Philosophical Implications*. Pp. xvii+611

The Harmonic Mind is a two-volume collection of 23 chapters authored by Smolensky, Legendre and their collaborators. Some of these are reprints or slight updates of previously published material, but approximately 3/4 of it has never appeared before. Every phonologist who takes at all seriously the notion that phonology is a branch of cognitive science should own a copy of this book.

Smolensky and Legendre (v.1 p.1) present a view of cognition that incorporates two distinct, but related levels of formal description: “the continuous, numerical lower-level description of brain”, characterized in terms of a connectionist network, and “the discrete, structural, higher-level description of mind”, characterized in terms of Optimality Theory (OT). The depth and breadth of the presentation is awe-inspiring, and quite likely intimidating in some places for any single reader. In their preface, Smolensky and Legendre characterize the intended audience as interdisciplinary groups of researchers in cognitive science, and a joint reading of this book by such a group would undoubtedly be extremely educational for all. Bridges between the disciplines are built by careful exposition of the fundamental ideas, and the experts in each area will be satisfied by the the wealth of insights, and the formal precision of their presentation. Since phonology is a central focus, this book provides phonologists with a unique opportunity to engage with both the research and the researchers in other realms of cognitive science.

There is furthermore much that phonologists (and other linguists) working on their own can gain from this book. The presentation of the overall cognitive framework in the first few chapters of *Volume 1: Cognitive Architecture* should be accessible to readers with little or no background in mathematics or computational modeling of cognition. This section should particularly appeal to phonologists curious about connectionism, but who are put off by reductionist applications that seek to eliminate the types of representations used in generative linguistics. Another key point of entry to the material in the first volume is Smolensky and Legendre’s “Formalizing the Principles II: Optimization and Grammar”, which discusses the relationship between connectionist optimization, grammatical well-formedness, and formal language theory. A further point of access to the model of cognition for linguists is found in Smolensky’s “Principle-Centered Connectionist and Generative-Linguistic Theories of Language”, in volume 2. The final paper in volume 1 “Optimality Theory: The Structure, Acquisition, and Use of Grammar”, by Smolensky, Legendre, and Bruce Tesar consists of an extremely lucid exposition of OT in the context of the overall generative enterprise, along with a set of cogent responses to common worries about the theory. Another chapter in volume 1 worth highlighting for its

*I would like to especially thank Paul Smolensky for sharing an advance copy of portions of the manuscript, and for comments on an earlier version of this review. Thanks also to the participants in Ling 730, Fall 2005 for discussion, and to Michael Becker, Shigeto Kawahara and John McCarthy for comments on the draft review.

interest to linguists is Smolensky’s “Optimization in Neural Networks: Harmony Maximization”. For anyone who would like to seriously explore the connectionist formalization of linguistic grammar, this chapter will serve as an essential guide to the relevant properties of different types of neural network.

Volume 2: Linguistic and Philosophical Implications is more directly targeted at a linguistic audience. For phonologists, Smolensky’s “Optimality in Phonology II: Harmonic Completeness, Local Conjunction, and Feature Domains” is a watershed publication. In it, Smolensky presents for the first time in print his already widely influential theory of local constraint conjunction, and also a theory of headed domains for phonological representations. Smolensky shows that headed domains allow a constraint-based theory to deal with phenomena in vowel harmony that can otherwise pose a considerable challenge, including directionality effects and other restrictions on the “trigger” of spreading (see e.g. Cole and Kisseberth 1995; McCarthy 2004 for related proposals in OT). This theory of assimilation has received relatively little attention in comparison with other OT accounts based on Agree constraints (e.g. Bakovic 2000) or on Alignment constraints (e.g. Kaun 2004). Smolensky’s successful headed domains reanalysis of Archangeli and Pulleyblank’s (1994) account of Lango ATR harmony, which involves a particularly complex interacting set of conditions on the direction, trigger and target of spreading, provides ample motivation for further exploration of this approach.

The local conjunction (henceforth LC) theory of constraint interaction is better known than the headed domains theory of representation, and is also much more controversial (see e.g. McCarthy 1999, 2003; Padgett 2002 for critiques; I draw on these below). Much of the controversy surrounds the worry that LC introduces into OT the power to generate unattested linguistic patterns that are ruled out by the strict domination property of ranked constraints. The relationship between LC and a version of OT that uses weighted constraints, as in OT’s predecessor, Harmonic Grammar (henceforth HG), is addressed in Legendre, Antonella Sorace, and Smolensky’s chapter “The Optimality Theory - Harmonic Grammar Connection”. The remainder of this review focuses on this relationship, which as Legendre et al. point out, can easily be misconstrued. Throughout, references to Legendre et al. are to the just-cited chapter, and those to Smolensky are to “Optimality in Phonology II”.

LC is an elaboration of OT originally proposed by Smolensky in a series of unpublished, but much-cited conference presentations. The literature on OT-LC is now quite large: along with the work cited in Smolensky’s chapter, an especially important further source is Ito and Mester’s (2003) monograph. Smolensky (v. 2, p. 43) defines local conjunction as follows:

- (1) Local conjunction within a domain D

*A& _{D} *B is violated if and only if a violation of *A and a (distinct) violation of *B both occur within a single domain of type D .

This operation of conjunction yields a new constraint that is separately rankable in the constraint hierarchy.

The original motivation for OT-LC, and the one discussed at length in Smolensky’s chapter, is to reduce complex markedness constraints to more basic primitives. A simple example that appears in his earlier unpublished work, but not in this chapter, is that of a constraint against voiced obstruent codas (see also Ito and Mester 2003). This can be expressed as the conjunction of two independently needed constraints, NOCODA (“assign a violation mark to a consonant in syllable final position”), and *VOICEOBS (“assign a violation mark to a voiced obstruent”). When these are conjoined in the domain of a segment, the resulting constraint penalizes voiced obstruent codas, but neither voiceless codas, nor voiced obstruents. If the conjoined constraint is ranked above the faithfulness constraint IDENT-VOICE (“assign a violation mark to a segment whose Input and Output voice specification differs”; McCarthy and Prince 1999), and the

component constraints are ranked beneath IDENT-VOICE, the result is coda devoicing:

(2) *Local conjunction analysis of final devoicing*

/bad/	NoCODA&*VOICEOBS	IDENT-VCE	NoCODA	*VOICEOBS
\mathbb{E} [bat]		*	*	*
bad	*!		*	**
pat		**!	*	
pad	*!	*	*	*

Obviously, the devoicing pattern could not be produced by just the unconjoined constraints in OT. Perhaps less obviously, it also cannot be produced by weighted constraint interaction in HG.

Before showing why HG cannot duplicate this effect of LC, it's worth first seeing how weighted versions of OT constraints can produce the devoicing pattern if we include *CODA-VOICE, which like the conjoined constraint, assigns violations to voiced codas. In HG, optimality is defined in terms of maximal numeric harmony: the optimal form has the highest summed weighted violation score. The HG tableau in (3) adds a row to show the weights of each constraint, and a column to show the harmony scores of each candidate. Following a convention introduced by Legendre et al., the violation marks are changed into the corresponding negative integers. Because *CODA-VOICE has greater weight than IDENT-VOICE, a candidate that incurs one violation of that constraint, like [bad], receives a greater weighted penalty ($2 \cdot (-1) = -2$) than a candidate that has a single violation of IDENT-VOICE, like [bat] ($1 \cdot (-1) = -1$). Since ([bat]) has the highest harmony in the candidate set, it is optimal.

(3) *Final devoicing in HG*

	2	1	
/bad/	*CODA-VOICE	IDENT-VOICE	
bad	-1		-2
\mathbb{E} [bat]		-1	-1
pat		-2	-2
pad	-1	-1	-3

To help show why HG cannot replicate the effect of local conjunction illustrated in (2), the table in (4) provides the violation profiles of the candidates using negative integers. I leave out the final, harmonically bounded, candidate from (2), whose violation marks on the basic constraints would be identical to [bat].

(4) *Violation scores of unconjoined constraints*

/bad/	IDENT-VOICE	NoCODA	*VOICEOBS
bat	-1	-1	-1
bad		-1	-2
pat	-2	-1	

The question is whether there exists some set of weights that will choose [bat] as optimal.

The answer is no. For [bat] to beat [bad], the weight of *VOICEOBS must be greater than that of IDENT-VOICE. And for [bat] to beat [pat] the reverse relationship must hold. These inequalities are inconsistent, just like the rankings would be. Both [bat] and [bad] violate NoCODA; that shared violation cannot affect the choice between them.

Legendre et al. (v.2, 348 ff.) provide a similar example of the inability of HG to handle a pattern analyzed in terms of OT-LC, and show that this inability generalizes to a set of cases discussed in Smolensky’s chapter. The example is that a ban on [x] cannot be reduced to independent constraints against velars and fricatives, when the inventory contains both velars and non-velar fricatives. The other cases are other OT-LC analyses of “basic inventories”. As they point out, in comparison with HG’s linear model (the harmony function is a linear equation), LC is a *superlinear* theory of constraint interaction.

In linguistic theory, greater power is of course a double-edged sword. The superlinearity of OT-LC also allows it to generate unattested linguistic patterns not generated by HG. One example, discussed first by Ito and Mester (1998), can be produced by conjoining NOCODA with IDENT-VOICE in the domain of the segment (thanks to Matt Wolf for bringing this to my attention).

(5) *Local conjunction analysis of initial devoicing*

/bad/	NOCODA& _{Seg} IDENT-VOICE	*VOICEOBS	NOCODA	IDENT-VCE
☞ [pad]		*	*	*
bat	*!	*	*	*
bad		**!	*	
pat	*!		*	*

The *VOICEOBS \gg IDENT-VCE ranking leads to devoicing, which is blocked by the dominant conjoined constraint when it would occur in coda position, where it violates both NOCODA and IDENT-VOICE. This pattern is referred to as a markedness reversal, since now the feature is protected in only the marked environment (see Lubowicz 2005 for further cases).

With just the unconjoined constraints in (5), this pattern cannot be produced by HG: [bat] and [pad] have identical violation marks, and no weighting will make [pad] solely optimal. In a full OT/HG constraint set that includes *CODA-VOICE, [bat] harmonically bounds [pad]. The fact that [pad] can be made optimal in OT-LC but not HG points to an important difference between the theories: pairwise harmonic bounding relationships from OT are preserved in a version of HG with positive weights (Prince 2002), but not in OT-LC.

Unattested systems can also be produced by conjoining markedness constraints. One such system devoices an obstruent if there is a coda anywhere in the word; this system emerges if we conjoin NOCODA and *VOICEOBS over the domain of the word:

(6) *Local conjunction analysis of long-distance “coda devoicing”*

/batalak/	NOCODA& _{wd} *VOICEOBS	IDENT-VCE	NOCODA	*VOICEOBS
batalak	*!		*	*
☞ [patalak]		*	*	

/batala/	NOCODA& _{wd} *VOICEOBS	IDENT-VCE	NOCODA	*VOICEOBS
☞ [batala]				*
patala		*!		

Again, this pattern cannot be replicated by HG, for the same reason it cannot do local coda devoicing with the unconjoined version of these constraints: the shared violation of NOCODA is irrelevant to the choice between [balatak] and [palatak].

This last case is an instance of the locality problem for LC. To deal with it, Lubowicz (2002) proposes to limit LC to conjunctions over the smallest shared domain of two constraints. However, LC also produces unattested patterns when constraints are conjoined over their smallest shared domain. To construct an example, we can add a constraint that forces place assimilation; for familiarity’s sake, I will use AGREE-PLACE (“assign a violation mark to every pair of adjacent of non-homorganic segments”). The first two tableaux show the effect of *CODA-VOICE and AGREE-PLACE being dominated by the relevant faithfulness constraints (IDENT-PLACE penalizes change in place of articulation): neither coda devoicing nor place assimilation usually applies. In the last tableau, we see that when a consonant with underlying voicing is in the context of a non-homorganic consonant, it devoices. Although place assimilation and coda devoicing are well attested, this cumulative interaction is to my knowledge undocumented.

(7) *Cumulative interaction of coda devoicing and place agreement in OT-LC*

/bad/	IDENT-VOICE	*CODA-VOICE
bat	*!	
☞ [bad]		*

/sat+ma/	IDENT-PLACE	AGREE-PLACE
☞ [satma]	*!	
sapma		*

/bad+ma/	*COD-VCE& _{Seg} AGR-PLC	ID-PLC	ID-VCE	*COD-VCE	AGR-PLC
☞ [batma]			*		*
babma		*!		*	
badma	*!			*	*

In HG, this unattested cumulative interaction is impossible with these constraints. If IDENT-VOICE has a greater weight than *CODA-VOICE, as needed to block devoicing in the first tableau, then [badma] will be optimal in the last tableau. Here, the shared violation of AGREE-PLACE is irrelevant to the choice between [batma] and [badma].

This is an instance of a co-relevance problem for LC. Empirically, place agreement is not relevant to coda voicing, but LC allows these independent factors to interact. Bakovic (2000) and Lubowicz (2005) propose restrictions on LC that are aimed at addressing this problem. Interestingly, the restriction that Lubowicz (2005) imposes on LC cumulative interactions emerges from the nature of HG cumulativity: for two markedness constraints to have a cumulative effect, they must be satisfied by violating a single other (faithfulness) constraint. For example, in Japanese loanword devoicing (Kawahara 2006), a voiced geminate is devoiced only in the presence of another voiced obstruent. In Pater (2009) this is analyzed as IDENT-VOICE being violated only if both OCP-VOICE and *VOICE-GEMINATE are at issue. Because this sort of asymmetric trade-off in violations is a prerequisite for cumulative effects in HG, it imposes inherent restrictions on such interactions that must be stipulated in the LC approach.

These instances of restrictiveness of HG relative to LC may come as a surprise to some readers, as they did to me and the other participants in a graduate seminar when we read Legendre et al. for the first time.¹ Two questions are immediately raised. The first is whether HG might actually be sufficiently restrictive to

¹It would be a mistake to claim that HG and LC are in a subset relation in terms of the languages they generate. Because LC requires that constraints be conjoined over some domain, there are instances of HG interaction that cannot be captured by LC (McCarthy 2002). For instance, Moreton and Smolensky (2002) claim that MAX and markedness constraints do not share a common domain, so they cannot be conjoined, but there is no reason that they could not participate in a gang effect.

serve as a viable alternative to OT as a theory of grammar. Along with Prince and Smolensky (1993/2004), Legendre et al. argue that this is not the case; see Pater (2009) for a detailed consideration of these arguments that yields a more positive conclusion for the prospects of HG. One point that is relevant to the current HG-LC comparison is that the unattested pattern that Legendre et al. show that HG can problematically generate can equally be produced by self-conjoining their “MAINSTRESSRIGHT” constraint. This constraint assigns a violation to each syllable separating a main stress from the right edge the word. The self-conjoined version would produce a “Power Hierarchy” of constraints, with one penalizing stress one syllable from the edge, another penalizing stress two syllables away, and so on. These constraints could be used to produce a stress window of any size, just as Legendre et al. show weightings of MAINSTRESSRIGHT can.

The other question is to what extent HG can, and should, deal with patterns that escape the original version of OT, and which have been attributed to LC. This is a very complex question, partly because some of the phenomena attributed to OT-LC may be best explained not as cumulative interaction at all (see recently McCarthy 2007 on chain shifts as well as Wolf 2008 on derived environment effects; cf. Lubowicz 2002 and Moreton and Smolensky 2002 in OT-LC). Here I will discuss only the examples already mentioned in this review.

First, whether complex markedness constraints like NOCODAVOICE above, and *[x] in Legendre *et al.* should be reduced to more basic constraints is a matter of some controversy. The idea that NOCODAVOICE is the sum of the effects of a constraint against codas and one against voiced obstruents harkens back in some ways to theories of prosodic licensing (e.g. Ito 1986; Goldsmith 1990; Lombardi 1991; Steriade 1995), which see contextual markedness as the inability of marked prosodic contexts to license marked segments. Much work in OT has questioned this approach, analyzing contextual markedness as the effect of rather specific, substantively motivated constraints (e.g. Pater 1999; Steriade 1999). The empirical motivation for these alternatives is that contextual markedness displays asymmetries that are not captured by prosodic licensing: the set of marked contexts is not the same for every marked segment, and markedness relationships between segments can be reversed across contexts (see Barnes 2006 for an extensive recent critique of prosodic licensing theory). Like prosodic licensing, OT-LC also fails to express these asymmetries.

Smolensky’s comparison of the OT-LC analysis of Lango ATR harmony with that of Archangeli and Pulleyblank (1994) brings up a concrete example of this issue (see Potts et al. 2009 for a reanalysis of the Lango data in HG). In Lango, only [+high] vowels trigger regressive ATR harmony. Smolensky analyzes this as the conjunction of the constraint against [-high, +ATR] vowels with the constraint demanding that heads be leftmost in the domain (v.2, 91). Leftward spreading from a nonhigh [+ATR] vowel would violate this constraint. Smolensky (v. 2, p. 138) points out that the rule-based analysis predicts that a rule that spreads ATR only from the marked vowel could equally exist, and cites Archangeli and Pulleyblank (1994) for the observation that the resulting pattern does not exist. It is clear, however, that other features do preferentially spread from a marked vowel: Kaun (2004) discusses a typology of rounding harmony in OT that instantiates just this observation.

Whether building a phonological theory with very specific phonetically grounded universal constraints is the right response to such asymmetries is of course also a matter of controversy (see Blevins, 2004; Hayes et al., 2004). However, the greater reductionism possible in an OT-LC account of contextual markedness does not seem to be a knockdown argument for it over OT and HG, especially since that reductionism is only obtained in a somewhat abstract sense, since Smolensky proposes no mechanism for the learning of conjoined constraints.

If the conjecture that HG can serve as a theory of grammar proves correct, the consequences for Smolen-

sky and Legendre’s integrated connectionist-symbolic theory of mind, and for the general relationship of grammar to theories of learning and processing, are quite deep. Smolensky and Legendre aim to provide an explicit mathematical characterization of the relationship between neural processing and higher-level symbolic computation, including linguistic grammar. They see HG as a bridge between these domains, with OT as the theory of grammar. However, this bridge is not yet fully built. Building on discussion in Prince and Smolensky (1993/2004: 236), Legendre et al. (v. 2, p. 347) explain the formal difficulties in constructing a connectionist harmony maximizing network that is guaranteed to behave as a strict domination hierarchy. HG grammars, on the other hand, have been realized in connectionist networks (see also Goldrick and Daland, this volume). In phonology, Legendre et al. present connectionist implementations of HG versions of Prince and Smolensky’s (1993/2004) analyses of Berber syllabification, while Melanie Soderstrom, Donald W. Mathis, and Smolensky’s chapter “Abstract Genomic Encoding of Universal Grammar in Optimality Theory” provides a connectionist learning algorithm for HG, along with a proposal for how universal constraints might be encoded in the genome (the title of the chapter is somewhat misleading in that the model uses weighted, rather than ranked constraints).

Smolensky, Legendre and their colleagues revolutionized linguistic theory by introducing optimization into generative grammar. As readers of this journal certainly know, the impact was especially strong in phonology. Along with some contemporaries (e.g. Goldsmith 1990 and papers in Goldsmith 1993), they can also be credited with introducing numerically weighted constraints into generative linguistics in their 1990 presentation of HG (an update of their 1990 work with Yoshiro Miyata: “The Interaction of Syntax and Smantics: A Harmonic Grammar Account of Split Intransitivity” appears in volume 1). Current probabilistic approaches to constraint interaction like the Maximum Entropy model of Goldwater and Johnson (2003) are formally stochastic versions of HG. Not only does *The Harmonic Mind* clearly elaborate the connectionist foundations for OT, but it also opens up vast new territory for phonological theory, for formal analysis of phonological systems as well for the study of learning and other types of processing.

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